# Competition, Product Safety, and Product Liability

**Online Appendix**

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In this online appendix, we first provide the detailed proofs of Lemmas 3 and 5 in the paper, and then discuss several alternative formulations to shed light on the robustness of our results. While our spatial model is relatively simple and intuitive to work with, it imposes specific structures that can be restrictive. We have also made other simplifying assumptions in the model to make the analysis tractable. In this online appendix, we consider in turn an alternative demand system, an alternative formulation on consumer precaution efforts, an alternative assumption on observability in period 2, and small consumer heterogeneity (small $t$). Our main insights concerning the relationship between competition and product liability remain valid in these alternative settings.

## A1. PROOFS OF LEMMAS 3 AND 5

In this section, we provide the detailed proofs for Lemma 3 and Lemma 5 in the paper.

**Proof of Lemma 3.** Suppose that firm 1 sells a low safety product while all the other firms sell the high safety product. Given that all products except for firm 1's product have the same quality, we focus on the symmetric pricing decision for any firm $j \neq 1$. Suppose that the prices are $q_j$, $j = 1, ..., N$. Then consumer $x_{1j}$ on $l_{1j}$, for $j \neq 1$, is indifferent between products 1 and $j$ if

$$V - x_{1j}t - q_1 - [(1-\alpha)\theta(D - \delta^*) + \phi(\delta^*)] = V - (1 - x_{1j})t - q_j - [(1-\alpha)\theta(d - \delta^*) + \phi(\delta^*)].$$

The demand for product 1 in Period 2 thus becomes

$$F_1(q_1,...,q_N) = \frac{2}{N(N-1)} \sum_{j\in\{2,..N\}} \max\left\{\min\left\{\frac{t-q_1+q_j-(1-\alpha)\theta z}{2t}, 1\right\}, 0\right\},$$

where we recall $z \equiv D - d$.

For any firm $j \neq 1$, it competes for two types of consumers: consumers located on $l_{1j}$, and consumers located on $l_{ij}$, for any $i \neq j$ and $i \neq 1$. For consumers located on $l_{1j}$, their demand for product $j$ is $\frac{2}{N(N-1)} \max\left\{\min\left\{\frac{t+q_1-q_j+(1-\alpha)\theta z}{2t}, 1\right\}, 0\right\}$. For consumers located on $l_{ij}$, given that products $j$ and $i$ have the same quality, their total demand for product $j$ is $\frac{2}{N(N-1)} \max\{\min\{\frac{t+q_i-q_j}{2t}, 1\}, 0\}$. Hence, the total demand for product $j \neq 1$ in Period 2 is

$$F_j(q_1,...,q_N) = \frac{2\max\left\{\min\left\{\frac{t+q_1-q_j+(1-\alpha)\theta z}{2t}, 1\right\}, 0\right\}}{N(N-1)} + \frac{2\sum_{i\neq j, i\neq 1, i\in\{1,..N\}} \max\left\{\min\left\{\frac{t+q_i-q_j}{2t}, 1\right\}, 0\right\}}{N(N-1)}.$$

First, assume that $F_1(q_1,...,q_N) \in (0,1)$ and $F_j(q_1,...,q_N) \in (0,1)$. That is, every firm has positive output. Then, firm 1's maximization problem is

$$\max_{q_1} \left\{ [q_1 - \alpha\theta(D-\delta^*)] \frac{2}{N(N-1)} \sum_{j\in\{2,..N\}} \frac{t-q_1+q_j-(1-\alpha)\theta z}{2t} \right\}.$$

The first order condition is

$$t - 2q_1 + q_j - (1-\alpha)\theta(D-d) + \alpha\theta(D-\delta^*) = 0.$$

For any $j \neq 1$,its maximization problem is

$$\max_{q_j} \left\{ [q_j - c - \alpha\theta(d - \delta^*)] \frac{2}{N(N-1)} \left[ \frac{t + q_1 - q_j + (1-\alpha)\theta z}{2t} + \sum_{i \neq j, i \neq 1, i \in \{1,..N\}} \frac{t + q_i - q_j}{2t} \right] \right\}.$$

In equilibrium, $q_i = q_j$ for $i \neq j, i \neq 1$. The first order condition leads to

$$(N-1)t - Nq_j + q_1 + (1-\alpha)\theta(D-d) + (N-1)\alpha\theta(d-\delta^*) + (N-1)c = 0.$$

Solving the above first order conditions, we have the optimal prices as

$$\begin{aligned} \tilde{q}_1 &= t + \alpha\theta(D - \delta^*) - \frac{N-1}{2N-1}(\theta z - c), \\ \tilde{q}_j &= t + c + \alpha\theta(d - \delta^*) + \frac{1}{2N-1}(\theta z - c) \text{ for any } j \neq 1. \end{aligned}$$

Notice that assumption A3 implies $t > \frac{N-1}{2N-1}(\theta z - c)$. Therefore, we have

$$\frac{t - \tilde{q}_1 + \tilde{q}_j - (1-\alpha)\theta z}{2t} = \frac{t - \frac{N-1}{2N-1}(\theta z - c)}{2t} \in (0,1).$$

Correspondingly, $F_1(\tilde{q}_1, ..., \tilde{q}_N) \in (0,1)$ and $F_j(\tilde{q}_1, ..., \tilde{q}_N) \in (0,1)$. And firm 1's profit in period 2 is

$$[\tilde{q}_1 - \alpha\theta(D - \delta^*)] \frac{2}{N} \frac{1}{N-1}(N-1) \frac{t - \frac{N-1}{2N-1}(\theta z - c)}{2t} = \frac{[t - \frac{N-1}{2N-1}(\theta z - c)]^2}{Nt}.$$

As a remark, if $t \leq \frac{N-1}{2N-1}(\theta z - c)$, however, $F_1(\tilde{q}_1, ..., \tilde{q}_N) = 0$. And firm 1's profit in period 2 is zero. ■

**Proof of Lemma 5.** Recall that $\Delta(N) = \frac{1}{N}[t - \frac{(t-t_2)^2}{t}]$, where $t_2 \equiv t_2(N) = \frac{N-1}{2N-1}(\theta z - c)$ strictly increases in $N$. $\Delta(N)$ can be re-written as

$$\Delta(N) = \frac{1}{N}\left\{2\frac{N-1}{2N-1}(\theta z - c) - \frac{1}{t}\left[\frac{N-1}{2N-1}(\theta z - c)\right]^2\right\}.$$

Differentiating $\Delta(N)$, we have, for $N \geq 2$:

$$\begin{aligned}
\Delta'(N) &= \frac{\theta z - c}{N^2(2N-1)^2}\{-2[(N-1)(2N-1) - N] + \frac{\theta z - c}{t}\frac{N-1}{2N-1}[(N-1)(2N-1) - 2N]\} \\
&< \frac{\theta z - c}{N^2(2N-1)^2}\{-2[(N-1)(2N-1) - N] + \frac{\theta z - c}{t}\frac{N-1}{2N-1}[(N-1)(2N-1) - N]\} \\
&= \frac{\theta z - c}{N^2(2N-1)^2}(-2 + \frac{\theta z - c}{t}\frac{N-1}{2N-1})(2N^2 - 4N + 1) \\
&< 0,
\end{aligned}$$

where the last inequality follows from $2N^2 - 4N + 1 > 0$ for $N \geq 2$ and $\frac{\theta z - c}{t}\frac{N-1}{2N-1} < 1$ when $t > t_2 = \frac{N-1}{2N-1}(\theta z - c)$. Therefore, $\Delta(N)$ strictly decreases in $N$. ■

## A2. ALTERNATIVE DEMAND WITH VARIABLE TOTAL OUTPUT

The demand in our main model, as in other spatial models of competition, has the restrictive feature that in equilibrium the market is fully covered with a fixed total output. We now explain that our main results concerning competition and product liability can also hold in an alternative model of consumer demand with variable total output (Daughety and

Reinganum, 2008b), which is derived from the qausi-linear quadratic utility function of a representative consumer.

As in the spatial model, there are $N$ firms, with each firm's product causing consumer harm with probability $\theta$. At the beginning of Period 1, a firm can choose to invest $k$, which enables it to produce a high-safety product in both periods at marginal cost $c \geq 0$. Without the investment, the product will be of low safety with zero marginal cost. After purchasing a product, a consumer can take precaution effort for that product. Without such effort, if a consumer is harmed, her damage is $d$ from a high-safety product and $D > d$ from a low-safety product. We define $z \equiv D - d$, and assume $c < \theta z$. For convenience, denote consumer belief about damage level from product $j$ as $B_j = \{D, d\}$.

Following Daughety and Reinganum (2008b), we consider the quasi-linear quadratic utility model where there is a single consumer who consumes $Q_j$ of product $j$, $j = 1, ...N$. When consuming each product, the consumer can take precaution effort, which reduces the damage from the particular product by $\delta \in [0, d)$ per unit. The unit precaution cost is $\phi(\delta)$, which is strictly increasing and convex, with $\phi(0) = 0$, $\phi'(0) = 0$, and $\phi'(d) > \theta$. With consumer precaution, the expected damage level from one unit of a high-safety product is $\theta(d - \delta)$, and the expected damage level from one unit of a low-safety product is $\theta(D - \delta)$.

The consumer's utility function is

$$U(Q_1, ..Q_N) = \sum_j [A - (1-\alpha)\theta(B_j - \delta_j) - \phi(\delta_j)]Q_j - \frac{1}{2}(\sum_j \beta Q_j^2 + \sum_j \sum_{i \neq j} \gamma Q_j Q_i),$$

where $\alpha$ is the liability level and $\gamma > 0$ is the degree of product substitution between any two products. Assume that $\beta > 0$ and $A$ is large enough such that there is positive demand for each product. Notice that $(1-\alpha)\theta(B_j - \delta_j) + \phi(\delta_j)$ is the sum of the consumer's per unit damage and precaution costs for product $j$.

The consumer with income $w$ maximizes her utility

$$\max_{Q_1,..Q_N} U(Q_1, ..Q_N) + w - \sum_j p_j Q_j.$$

The above maximization problem leads to the inverse demand function

$$p_j(Q_1, ..Q_N) = A - \theta(1-\alpha)(B_j - \delta_j) - \phi(\delta_j) - \beta Q_j - \gamma \sum_{i \neq j} Q_i.$$

Solving for the ordinary demand function, we have

$$\begin{aligned} & Q_j(p_1, ..p_N; B_1, ...B_N) \\ = \quad & A' - b[(1-\alpha)\theta(B_j - \delta_j) + \phi(\delta_j)] + g\sum_{i \neq j}[(1-\alpha)\theta(B_i - \delta_i) + \phi(\delta_i)] - bp_j + g\sum_{i \neq j} p_i \ , \end{aligned}$$

where $A' = \frac{A}{\beta+(N-1)\gamma}$, $b = \frac{[\beta+(N-2)\gamma]}{(\beta-\gamma)[\beta+(N-1)\gamma]}$, and $g = \frac{\gamma}{(\beta-\gamma)[\beta+(N-1)\gamma]}$.

Given the additive nature, the consumer's precaution effort would be the same for all products. Following the analysis under the spatial model, the consumer's optimal precaution is $\delta_j = \delta^* \equiv \delta(\alpha)$ for $j = 1, ...N$. The following lemma is the same as that under the spatial model.

**Lemma A 1** *Given the firms' safety investments, the consumer's precaution effort* $\delta(\alpha)$

*and total welfare are higher when product liability $\alpha$ is lower.*

The full-fledged analysis based on the general demand function becomes rather complicated. For simplicity, we consider two special scenarios investigating the relationship between liability and the different measures of competition, $N$ and $\gamma$, respectively.

### A2.1 PRODUCT LIABILITY AND THE NUMBER OF COMPETITORS

In this subsection, assume that $\beta = 2$ and $\gamma = 1$. Then the demand function becomes

$$\begin{aligned} Q_j(p_1, ..p_N; B_1, ...B_N) &= \frac{A}{N+1} - \frac{N}{N+1}[(1-\alpha)\theta(B_j - \delta^*) + \phi(\delta^*)] \\ &+ \frac{1}{N+1}\sum_{i \neq j}[(1-\alpha)\theta(B_i - \delta^*) + \phi(\delta^*)] \\ &- \frac{N}{N+1}p_j + \frac{1}{N+1}\sum_{i \neq j} p_i. \end{aligned}$$

**Analysis for Period 2**

We focus on the symmetric equilibrium where all firms invest in product safety and charge the same price. As long as all firms have positive outputs in period 1, the consumer and firms in period 2 observe each firm's product safety. As in the paper, we also assume that, if in some off-equilibrium path a firm has zero output in period 1, the consumer and other firms in period 2 will hold the same belief as the consumer in period 1. Along the equilibrium path, suppose that the prices are $q_j$, $j = 1, ..., N$ in period 2. The demand for any product $j$ would be $Q_j(q_1, ..q_N; B_1..B_N = d)$. Firm $j$ chooses price $q_j$ to maximize its profit

$$\max_{q_j}[q_j - c - \alpha\theta(d-\delta^*)]Q_j(q_1,..q_N;B_1..B_N=d).$$

The first order condition is

$$Q_j(q_1,..q_N;B_1..B_N=d) - \frac{N}{N+1}[q_j - c - \alpha\theta(d-\delta^*)] = 0.$$

Then the optimal price satisfies

$$q_j = q^* = \alpha\theta(d-\delta^*) + \frac{1}{N+1}[A - \theta(d-\delta^*) - \phi(\delta^*) + Nc].$$

Correspondingly, each firm's optimal output is

$$Q^* = \frac{N}{(N+1)^2}[A - \theta(d-\delta^*) - \phi(\delta^*) - c].$$

Thus, each firm's profit in period 2 is

$$\Pi^* = Q^*[q^* - c - \alpha\theta(d-\delta^*)] = \frac{N}{(N+1)^3}[A - \theta(d-\delta^*) - \phi(\delta^*) - c]^2,$$

which does not depend on liability $\alpha$.

In the high-safety equilibrium, consumer utility in period 2 is

$$
\begin{aligned}
U(Q^*, ..Q^*) + w - \sum_j q^* Q^* &= N[A - (1-\alpha)\theta(d-\delta^*) - \phi(\delta^*)]Q^* - \frac{1}{2}(2NQ^{*2} + N(N-1)Q^{*2}) + w \\
&\quad -N\{\alpha\theta(d-\delta^*) + \frac{1}{N+1}[A - \theta(d-\delta^*) - \phi(\delta^*) + Nc]\}Q^* \\
&= N[A - \theta(d-\delta^*) - \phi(\delta^*)]Q^* - \frac{1}{2}(2NQ^{*2} + N(N-1)Q^{*2}) + w \\
&\quad -N\{\frac{1}{N+1}[A - \theta(d-\delta^*) - \phi(\delta^*) + Nc]\}Q^*,
\end{aligned}
$$

which does not depend on liability $\alpha$ either. To summarize, we have the following lemma.

**Lemma A 2** *Suppose that all firms choose high safety. In period 2, there is a unique symmetric equilibrium where each firm sets price $q^*$, sells $Q^*$ units of output, and earns a profit of $\Pi^* = \frac{N}{(N+1)^3}[A - \theta(d-\delta^*) - \phi(\delta^*) - c]^2$. In the high-safety equilibrium with all firms charging $q^*$, both firm profit and consumer utility are independent of $\alpha$.*

If one firm, say firm 1, has deviated to $I = 0$, as long as it has positive output in period 1, it will be known as the low-safety firm in period 2. The demand for product 1 in Period 2 is thus

$$
\begin{aligned}
Q_1(q_1, ...q_N; B_1 &= D, B_{j\neq 1} = d) = \frac{1}{N+1}[A - \phi(\delta^*) - \theta(d-\delta^*) - N\theta z - \\
&\quad N(q_1 - \alpha\theta(D-\delta^*)) + \sum_{j\neq 1}(q_j - \alpha\theta(d-\delta^*))].
\end{aligned}
$$

Firm 1 chooses $q_1$ to maximize its profit in Period 2

$$\max_{q_1}[q_1 - \alpha\theta(D - \delta^*)]Q_1(q_1, ...q_N; B_1 = D, B_{j\neq 1} = d).$$

The first order condition is

$$A - \phi(\delta^*) - \theta(d - \delta^*) - N\theta z - 2N(q_1 - \alpha\theta(D - \delta^*)) + \sum_{j\neq 1}(q_j - \alpha\theta(d - \delta^*)) = 0.$$

The total demand for product $j \neq 1$ in Period 2 is

$$\begin{aligned} Q_j(q_1, ...q_N; B_1 &= D, B_{i\neq 1} = d) = \frac{1}{N+1}[A - \phi(\delta^*) - \theta(d - \delta^*) + \theta z - \\ & N(q_j - \alpha\theta(d - \delta^*)) + (q_1 - \alpha\theta(D - \delta^*)) + \sum_{i\neq 1, i\neq j}(q_i - \alpha\theta(d - \delta^*))]. \end{aligned}$$

Firm $j \neq 1$ chooses $q_j$ to maximize its profit in Period 2:

$$\max_{q_j}[q_j - c - \alpha\theta(d - \delta^*)]\, Q_j(q_1, ...q_N; B_1 = D, B_{j\neq 1} = d).$$

Given $q_j = q_i$ for any $i \neq 1$, the first order condition is

$$A - \phi(\delta^*) - \theta(d - \delta^*) + \theta z + Nc - (N+2)(q_j - \alpha\theta(d - \delta^*)) + (q_1 - \alpha\theta(D - \delta^*)) = 0.$$

The above first order conditions lead to the optimal prices

$$\widetilde{q}_1 = \alpha\theta(D-\delta^*) + \frac{1}{N+1}[A - \phi(\delta^*) - \theta(d-\delta^*) - c - \frac{N^2+N+1}{2N+1}(\theta z - c)].$$

$$\widetilde{q}_j = \alpha\theta(d-\delta^*) + \frac{1}{N+1}[A - \phi(\delta^*) - \theta(d-\delta^*) + \frac{2N^2}{2N+1}c - \frac{N}{2N+1}\theta z].$$

For simplicity, we focus on the range of $N$ such that $\widetilde{q}_1 - \alpha\theta(D-\delta^*) > 0$. Accordingly, firm 1's output is positive and its profit in period 2 is

$$\Pi_2^d = \frac{N}{(N+1)^3}[A - \theta(d-\delta^*) - \phi(\delta^*) - c - \frac{N^2+N+1}{2N+1}(\theta z - c)]^2.$$

Therefore, firm 1's reputation loss in period 2 is

$$\begin{aligned}\Delta(N) \quad \equiv \quad & \frac{N}{(N+1)^3}[A - \theta(d-\delta^*) - \phi(\delta^*) - c]^2 \\ & - \frac{N}{(N+1)^3}[A - \theta(d-\delta^*) - \phi(\delta^*) - c - \frac{N^2+N+1}{2N+1}(\theta z - c)]^2,\end{aligned}$$

which is independent of liability $\alpha$.

**Lemma A 3** $\Delta(N)$ *strictly decreases in* $N$.

**Proof.** For simplicity, denote $X = A - \theta(d-\delta^*) - \phi(\delta^*) - c$, and $Y(N) = \frac{N^2+N+1}{2N+1}(\theta z - c)$. Given $\theta z > c$, $Y(N) > 0$. Then we have

$$\Delta(N) \equiv \frac{N}{(N+1)^3}X^2 - \frac{N}{(N+1)^3}[X - Y(N)]^2,$$

Note that $\frac{d\Delta(N)}{dN} = -\frac{1}{(N+1)^3}\{\frac{2N-1}{N+1}[X^2 - (X - Y(N))^2] - \frac{2N^3+2N^2-N}{2N+1}2(\theta z - c)[X - Y(N)]\}$.

It can be re-written as

$$\frac{d\Delta(N)}{dN} = -\frac{1}{(N+1)^3}\{\frac{2N-1}{N+1}\frac{N^2+N+1}{2N+1}(\theta z - c)[2X - Y(N)] - \frac{2N^3+2N^2-N}{2N+1}(\theta z - c)[2X - 2Y(N)]\}.$$

For any $N \geq 2$, it can be verified that $\frac{2N-1}{N+1}\frac{N^2+N+1}{2N+1} > \frac{2N^3+2N^2-N}{2N+1}$. Additionally, $2X - Y(N) > 2X - 2Y(N) > 0$. Therefore, we have $\frac{d\Delta(N)}{dN} < 0$. ■

**Analysis for Period 1**

In period 1, the consumer only observes firms' prices but not product safety. Similar to the analysis in the paper, we assume that the consumer will believe a firm to have chosen product safety optimally, given its deviating price and the other firms' equilibrium prices. In particular, denote $p^e$ as the equilibrium price in period 1 in a symmetric high-safety equilibrium, and $B_j(p_j, p^e) = \{d, D\}$ as consumer belief about product $j$'s safety when firm $j$ charges $p_j$ while all other firms charge the same equilibrium price. Given any price $p_j \neq p^e$ and the prior that all products have high safety, if choosing high safety indeed can generate total profit (in two periods) for firm $j$ no less than choosing low safety, then consumer belief should be $B_j(p_j, p^e) = d$. In contrast, given $p_j \neq p^e$ and the prior that all products have high safety, if choosing low safety can generate strictly more profit for firm $j$ than choosing high safety, then consumer belief should be $B_j(p_j, p^e) = D$.

Now we characterize the set of consumer belief. Suppose that, in period 1, firm $j$ charges $p_j$ while all other firms charge the equilibrium price $p^e$. As in the paper, if firm $j$ charges a very high price and therefore has zero output, we assume that the consumer believes product $j$ to have low safety. Accordingly, the consumer and other firms in period 2 hold the same belief that product $j$ has low safety. If firm $j$ instead has positive output in period 1, its true safety will be revealed in period 2. Thus, given the prior that all products have high safety, firm $j$'s total profit in two periods by choosing high safety is

$$-k + [p_j - c - \alpha\theta(d - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_1...B_N = d) + \Pi^*.$$

Similarly, firm $j$'s total profit in two periods by choosing low safety is

$$[p_j - \alpha\theta(D - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_1...B_N = d) + \Pi_2^d.$$

Therefore, given $p_j$ and $p^e$, choosing high safety can bring larger profit to firm $j$ if and only if

$$\Delta(N) \geq k - (\alpha\theta z - c)Q_j(p_j, p_{-j} = p^e; B_1...B_N = d). \tag{A1}$$

Notice that the right hand side of condition (A1) is strictly increasing in $p_j$ when $\alpha\theta z - c > 0$ while strictly decreasing in $p_j$ when $\alpha\theta z - c < 0$. Similar to the analysis under the spatial model, when $\alpha\theta z - c \neq 0$, define $p'$ as the price level $p_j$ such that condition (A1) holds with equality. Thus, consumer belief takes the following form:

(1) If $\alpha\theta z - c = 0$, then $B_j(p_j, p^e) = d$ if $\Delta(N) \geq k$ while $B_j(p_j, p^e) = D$ if $\Delta(N) < k$;

(2) If $\alpha\theta z - c < 0$, then $B_j(p_j, p^e) = d$ if $p_j \geq p'$ while $B_j(p_j, p^e) = D$ if $p_j < p'$;

(3) If $\alpha\theta z - c > 0$, then $B_j(p_j, p^e) = d$ if $p_j \leq p'$ while $B_j(p_j, p^e) = D$ if $p_j > p'$.

Define $\alpha^N$ such that $\Delta(N) = k - (\alpha^N\theta z - c)Q_j(p_j = p_{-j} = p^e; B_1...B_N = d)$. It is easy to verify that $p' \neq p^e$ if $\alpha > \alpha^N$,while $p' = p^e$ if $\alpha = \alpha^N$. Similar to the analysis under the spatial model, it can be shown that high-safety equilibrium does not exist if $\alpha < \alpha^N$. Furthermore, if $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, we always have $\Delta(N) \geq k$ and therefore $B_j(p_j, p^e) = d$.

Now suppose that $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta z - c \neq 0$. The above analysis implies that, if firm $j$ charges a price which is in a small neighbourhood of $p^e$, the consumer believes that product $j$ has high safety. Thus, in any high-safety equilibrium, the optimal price for firm $j$ solves the following problem:

$$\max_{p_j}[p_j - c - \alpha\theta(d - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_1...B_N = d).$$

We then have the optimal price for firm $j$ satisfying

$$p^e = p^* = \alpha\theta(d - \delta^*) + \frac{1}{N+1}[A - \theta(d - \delta^*) - \phi(\delta^*) + Nc].$$

Notice that, if firm $j$ chooses a price much higher or lower than $p^*$ so that consumers would believe that it has deviated to low safety, then its profit in period 1 would be lower. In sum, whenever $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta z - c \neq 0$, if the high-safety equilibrium exists, it must be unique with all firms charging $p^*$ and earns $\Pi^* - k$ in period 1.

However, when $\alpha = \alpha^N$ and $\alpha\theta z - c \neq 0$, without further refinement, we can have a continuum of high-safety equilibria. We restrict our equilibrium concept to "stable equilibrium" as defined in the paper. In what follows, we shall focus on the stable equilibrium when there exist multiple equilibria. Therefore, for any $\alpha \geq \alpha^N$, the unique equilibrium price in period 1 is $p^e = p^*$.

Combining the above analysis for period 1 with that for period 2, we see that when the (stable) high-safety equilibrium exists, it is unique with all firms charging $p^* = q^*$ in both periods and earning total profits $2\Pi^* - k$ in two periods together. We next characterize the conditions for the existence of the equilibrium. Recall that the high-safety equilibrium does not exist if $\alpha < \alpha^N$. Suppose that $\alpha \geq \alpha^N$. At the proposed equilibrium where all firms choose high safety, if a firm, say firm 1, charges a price leading the consumer to believe that it has high safety, given $\alpha \geq \alpha^N$, our earlier analysis suggests that the firm should have no incentive to deviate to low safety. Thus, if firm 1 indeed deviates to low safety, it would choose a price leading the consumer to believe that it has low safety. Then similar to the analysis in the paper, no matter whether firm 1 has positive or zero output in period 1, the consumer in period 2 will hold the correct belief that product 1 has low safety. Given that all other firms charge $p^*$, the demand for product 1 in period 1 would be

$$Q_1(p_1, p_{j\neq 1} = p^*; B_1 = D, B_{j\neq 1} = d) = \frac{1}{N+1}[A - \phi(\delta^*) - \theta(d - \delta^*) - N\theta z - Np_1 + (N-1)p^*].$$

Ignoring the constraint $B_1(p_1, p^*) = D$, firm 1's optimal price after deviation in period 1 therefore solves:

$$\max_{p_1} [p_1 - \alpha\theta(D - \delta^*)] Q_1(p_1, p_{j\neq 1} = p^*; B_1 = D, B_{j\neq 1} = d).$$

The optimal deviating price is $\tilde{p}_1 = \frac{1}{N+1}[A - \theta(d-\delta^*) - \phi(\delta^*) + Nc] - \frac{\theta z - c}{2} + \alpha\theta(D-\delta^*)$, if and only if $B_1(\tilde{p}_1, p^*) = D$. If $B_1(\tilde{p}_1, p^*) = D$, firm 1's deviating profit in period 1 is

$$\Pi_1^d = \frac{N}{N+1}\{\frac{1}{N+1}[A - \theta(d-\delta^*) - \phi(\delta^*) + Nc] - \frac{\theta z - c}{2}\}^2,$$

which does not depend on $\alpha$.

Accordingly, if $B_1(\tilde{p}_1, p^*) = D$, the difference between firm 1's equilibrium profit and deviation profit in two periods is $\Pi^* - \Pi_1^d + \Delta(N) - k$, which does not depend on $\alpha$. In this scenario, the high-safety equilibrium exists as long as $\Pi^* - \Pi_1^d + \Delta(N) \geq k$.

To summarize, we have derived a sufficient condition for the existence of the high-safety equilibrium: $\alpha \geq \alpha^N$, and $\Pi^* - \Pi_1^d + \Delta(N) \geq k$.

**Proposition A 1** *There exists a unique equilibrium where all firms produce the high-safety product and charge $p^*$ in both periods if*

$$\alpha \geq \alpha^N, \ \textit{and} \ (\Pi^* - \Pi_1^d) + \Delta(N) \geq k.$$

**Optimal Liability**

According to Lemma A2, in the high-safety equilibrium with all firms charging $p^*$, both equilibrium profit and consumer utility do not depend on the liability level $\alpha$. Therefore, $\alpha$ affects social welfare only by changing firms' investment incentives. Notice that $\Pi^* - \Pi_1^d + \Delta(N)$ does not depend on $\alpha$ either. Now suppose that $\Pi^* - \Pi_1^d + \Delta(N) \geq k$. Define the socially optimal liability as $\alpha^*$. Lemma A1 and Proposition A1 imply that, whenever

$\alpha^* \in (0,1]$, we have $\alpha^* = \alpha^N$. Recall that $\alpha^N$ satisfies $\Delta(N) = k - (\alpha^N \theta z - c) Q_j(p_j = p_{-j} = p^*; B_1 ... B_N = d)$. Therefore, whenever $\alpha^* \in (0,1]$, it satisfies

$$\Delta(N) = k - \frac{N}{(N+1)^2}[A - \theta(d - \delta^*) - \phi(\delta^*) - c][\alpha^* \theta z - c]. \tag{A2}$$

The right hand side of condition (A2) decreases in $N$ when $\alpha^* \theta z - c < 0$ and increases in $N$ when $\alpha^* \theta z - c > 0$. Additionally, the reputation loss in period 2, $\Delta(N)$, always decreases in $N$. Notice that these two effects are similar to those under our spatial model in the paper. Thus, similar to the analysis in the paper, the following non-monotonicity result holds.

**Proposition A 2** *Suppose that* $\alpha^* > 0$ *for* $N \in [N_1, N_2]$, *with* $2 \leq N_1 < N_2 \leq \overline{N}$. *There exists some* $\widehat{N} \in [N_1, N_2]$ *such that, for any* $N \in [N_1, N_2]$, *the optimal liability* $\alpha^*$ *decreases in* $N$ *for* $N < \widehat{N}$ *and increases in* $N$ *for* $N > \widehat{N}$.

### A2.2 PRODUCT LIABILITY AND PRODUCT DIFFERENTIATION

In this subsection, we focus on the relationship between the socially optimal liability and competition as measured by the degree of product substitution $\gamma$. For simplicity, assume that $N = 2, \beta = 1$,and $\gamma \in (0,1)$. Then the demand function for product $j$ becomes

$$\begin{aligned} Q_j(p_j, p_{-j}; B_j, B_{-j}) \quad = \quad & \frac{A}{1+\gamma} - \frac{1}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(B_j - \delta^*) + \phi(\delta^*)] \\ & + \frac{\gamma}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(B_{-j} - \delta^*) + \phi(\delta^*)] \\ & - \frac{1}{(1-\gamma)(1+\gamma)} p_j + \frac{\gamma}{(1-\gamma)(1+\gamma)} p_{-j}. \end{aligned}$$

**Analysis for Period 2**

We focus on the symmetric equilibrium where all firms invest in product safety and charge the same price. As long as all firms have positive outputs in period 1, the consumer and firms in period 2 observe each firm's product safety. As in the paper, we also assume that, if in some off-equilibrium path a firm has zero output in period 1, the consumer and other firms in period 2 will hold the same belief as the consumer in period 1. Along the equilibrium path, The demand for any product $j$ is $Q_j(q_j, q_{-j}; B_j = B_{-j} = d)$. In period 2, firm $j$ chooses price $q_j$ to maximize its profit

$$\max_{q_j}[q_j - c - \alpha\theta(d - \delta^*)]Q_j(q_j, q_{-j}; B_j = B_{-j} = d).$$

The first order condition is

$$Q_j(q_j, q_{-j}; B_j = B_{-j} = d) - \frac{1}{(1-\gamma)(1+\gamma)}[q_j - c - \alpha\theta(d - \delta^*)] = 0.$$

Then the optimal price satisfies

$$q_j = q^* = c + \alpha\theta(d - \delta^*) + \frac{1-\gamma}{2-\gamma}[A - c - \theta(d - \delta^*) - \phi(\delta^*)].$$

Correspondingly, each firm's optimal output is

$$Q^* = \frac{1}{(1+\gamma)(2-\gamma)}[A - c - \theta(d - \delta^*) - \phi(\delta^*)].$$

And each firm's profit in period 2 is

$$\Pi^* = Q^*[p^* - c - \alpha\theta(d - \delta^*)] = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2}[A - \theta(d-\delta^*) - \phi(\delta^*) - c]^2,$$

which does not depend on liability $\alpha$.

In the high-safety equilibrium, consumer utility in period 2 is

$$\begin{aligned} U(Q^*, ..Q^*) + w - \sum_j p^* Q^* &= 2[A - (1-\alpha)\theta(d-\delta^*) - \phi(\delta^*)]Q^* - \frac{1}{2}(2Q^{*2} + 2\gamma Q^{*2}) + w \\ &\quad -2\{c + \alpha\theta(d-\delta^*) + \frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)]\}Q^* \\ &= 2[A - \theta(d-\delta^*) - \phi(\delta^*)]Q^* - \frac{1}{2}(2Q^{*2} + 2\gamma Q^{*2}) + w \\ &\quad -2\{c + \frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)]\}Q^*, \end{aligned}$$

which does not depend on liability $\alpha$ either. To summarize, we have the following lemma.

**Lemma A 4** *Suppose that all firms choose high safety. In period 2, there is a unique symmetric equilibrium where each firm sets price $q^*$, sells $Q^*$ units of output, and earns a profit of $\Pi^* = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2}[A - \theta(d-\delta^*) - \phi(\delta^*) - c]^2$. In the high-safety equilibrium with all firms charging $q^*$, both firm profit and consumer utility are independent of $\alpha$.*

If one firm, say firm 1, has deviated to $I = 0$, as long as it has positive output in period 1, it will be known as the low-safety firm in period 2. Suppose that the prices are $q_1$ and $q_2$.

The demand for product 1 in Period 2 is thus

$$
\begin{aligned}
Q_1(q_1, q_2; B_1 &= D, B_2 = d) = \frac{A}{1+\gamma} - \frac{1}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(D-\delta^*) + \phi(\delta^*)] \\
&+\frac{\gamma}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)] \\
&-\frac{1}{(1-\gamma)(1+\gamma)}q_1 + \frac{\gamma}{(1-\gamma)(1+\gamma)}q_2.
\end{aligned}
$$

Firm 1 chooses $q_1$ to maximize its profit in Period 2

$$
\max_{q_1}[q_1 - \alpha\theta(D-\delta^*)]Q_1(q_1, q_2; B_1 = D, B_2 = d).
$$

The first order condition is

$$
\begin{aligned}
0 &= (1-\gamma)A - [\theta(D-\delta^*) + \phi(\delta^*)] + \gamma[\theta(d-\delta^*) + \phi(\delta^*)] \\
&\quad -2[q_1 - \alpha\theta(D-\delta^*)] + \gamma[q_2 - \alpha\theta(d-\delta^*)].
\end{aligned}
$$

The total demand for product 2 in Period 2 is

$$
\begin{aligned}
Q_2(q_1, q_2; B_1 &= D, B_2 = d) = \frac{A}{1+\gamma} - \frac{1}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)] \\
&+\frac{\gamma}{(1-\gamma)(1+\gamma)}[(1-\alpha)\theta(D-\delta^*) + \phi(\delta^*)] \\
&-\frac{1}{(1-\gamma)(1+\gamma)}q_2 + \frac{\gamma}{(1-\gamma)(1+\gamma)}q_1.
\end{aligned}
$$

Firm 2 chooses $q_2$ to maximize its profit in Period 2:

$$\max_{q_2} \left[q_2 - c - \alpha\theta(d - \delta^*)\right] Q_2(q_1, q_2; B_1 = D, B_2 = d).$$

The first order condition is

$$\begin{aligned} 0 &= (1-\gamma)A + c - [\theta(d-\delta^*) + \phi(\delta^*)] + \gamma[\theta(D-\delta^*) + \phi(\delta^*)] \\ & \quad -2[q_2 - \alpha\theta(d-\delta^*)] + \gamma[q_1 - \alpha\theta(D-\delta^*)]. \end{aligned}$$

The above first order conditions lead to the optimal price for Firm 1

$$\widetilde{q}_1 = \alpha\theta(D-\delta^*) + \frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)] - \frac{2-\gamma^2}{4-\gamma^2}(\theta z - c).$$

Accordingly, firm 1's deviation profit in period 2 is

$$\Pi_2^d = \frac{1}{(1-\gamma)(1+\gamma)}\{\frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)] - \frac{2-\gamma^2}{4-\gamma^2}(\theta z - c)\}^2.$$

Therefore, firm 1's reputation loss in period 2 is

$$\begin{aligned} \Delta &\equiv \Pi^* - \Pi_2^d = \frac{1}{(1-\gamma)(1+\gamma)}\{\frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)]\}^2 \\ & \quad -\frac{1}{(1-\gamma)(1+\gamma)}\{\frac{1-\gamma}{2-\gamma}[A - c - \theta(d-\delta^*) - \phi(\delta^*)] - \frac{2-\gamma^2}{4-\gamma^2}(\theta z - c)\}^2, \end{aligned}$$

which is independent of liability $\alpha$.

**Analysis for Period 1**

In period 1, the consumer only observes firms' prices but not product safety. Similar to the analysis in the paper, we assume that the consumer will believe a firm to have chosen product safety optimally, given its deviating price and the other firms' equilibrium prices. In particular, denote $p^e$ as the equilibrium price in period 1 in a symmetric high-safety equilibrium, and $B_j(p_j, p^e) = \{d, D\}$ as consumer belief about product $j$'s safety when firm $j$ charges $p_j$ while the other firm charges the equilibrium price. Given any price $p_j \neq p^e$ and the prior that all products have high safety, if choosing high safety indeed can generate total profit (in two periods) for firm $j$ no less than choosing low safety, then consumer belief should be $B_j(p_j, p^e) = d$. In contrast, given $p_j \neq p^e$ and the prior that all products have high safety, if choosing low safety can generate strictly more profit for firm $j$ than choosing high safety, then consumer belief should be $B_j(p_j, p^e) = D$.

Now we characterize the set of consumer belief. Suppose that, in period 1, firm $j$ charges $p_j$ while the other firm charges the equilibrium price $p^e$. As in the paper, if firm $j$ charges a very high price and therefore has zero output, we assume that the consumer believes product $j$ to have low safety. Accordingly, the consumer and other firms in period 2 hold the same belief that product $j$ has low safety. If firm $j$ instead has positive output in period 1, its true safety will be revealed in period 2. Given the prior that all products have high safety, firm $j$'s total profit in two periods by choosing high safety is

$$-k + [p_j - c - \alpha\theta(d - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_j = B_{-j} = d) + \Pi^*.$$

Similarly, firm $j$'s total profit in two periods by choosing low safety is

$$[p_j - \alpha\theta(D - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_j = B_{-j} = d) + \Pi_2^d.$$

Therefore, given $p_j$ and $p^e$, choosing high safety can bring larger profit to firm $j$ if and only if

$$\Delta \geq k - (\alpha\theta z - c)Q_j(p_j, p_{-j} = p^e; B_j = B_{-j} = d). \tag{A3}$$

Notice that the right hand side of condition (A3) is strictly increasing in $p_j$ when $\alpha\theta z - c > 0$ while strictly decreasing in $p_j$ when $\alpha\theta z - c < 0$. Similar to the analysis under the spatial model, when $\alpha\theta z - c \neq 0$, define $p'$ as the price level $p_j$ such that condition (A3) holds with equality. Thus, consumer belief takes the following form:

(1) If $\alpha\theta z - c = 0$, then $B_j(p_j, p^e) = d$ if $\Delta \geq k$ while $B_j(p_j, p^e) = D$ if $\Delta < k$;

(2) If $\alpha\theta z - c < 0$, then $B_j(p_j, p^e) = d$ if $p_j \geq p'$ while $B_j(p_j, p^e) = D$ if $p_j < p'$;

(3) If $\alpha\theta z - c > 0$, then $B_j(p_j, p^e) = d$ if $p_j \leq p'$ while $B_j(p_j, p^e) = D$ if $p_j > p'$.

Define $\alpha^\gamma$ such that $\Delta = k - (\alpha^\gamma\theta z - c)Q_j(p_j = p_{-j} = p^e; B_j = B_{-j} = d)$. It is easy to verify that $p' \neq p^e$ if $\alpha > \alpha^\gamma$,while $p' = p^e$ if $\alpha = \alpha^\gamma$. Similar to the analysis under the spatial model, it can be shown that the high-safety equilibrium does not exist if $\alpha < \alpha^\gamma$. Furthermore, if $\alpha \geq \alpha^\gamma$ and $\alpha\theta z - c = 0$, we always have $\Delta \geq k$ and therefore $B_j(p_j, p^e) = d$.

Now suppose that $\alpha \geq \alpha^\gamma$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^\gamma$ and $\alpha\theta z - c \neq 0$. The above analysis implies that, if firm $j$ charges a price which is in a small neighbourhood of $p^e$, the consumer believes that product $j$ has high safety. Thus, in any high-safety equilibrium in period 1, the optimal price for firm $j$ solves the following problem:

$$\max_{p_j}[p_j - c - \alpha\theta(d - \delta^*)]Q_j(p_j, p_{-j} = p^e; B_j = B_{-j} = d).$$

We then have the optimal price for firm $j$ satisfying

$$p^e = p^* = c + \alpha\theta(d - \delta^*) + \frac{1-\gamma}{2-\gamma}[A - c - \theta(d - \delta^*) - \phi(\delta^*)].$$

Notice that, if firm $j$ chooses a price much higher or lower than $p^*$ so that consumers would believe that it has deviated to low safety, then its profit in period 1 would be lower. In sum, whenever $\alpha \geq \alpha^\gamma$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^\gamma$ and $\alpha\theta z - c \neq 0$, if a high-safety equilibrium exists, it must be unique with all firms charging $p^*$ and earns $\Pi^* - k$ in period 1.

However, when $\alpha = \alpha^\gamma$ and $\alpha\theta z - c \neq 0$, without further refinement, we can have a continuum of high-safety equilibria. We restrict our equilibrium concept to "stable equilibrium" as defined in the paper. Then, the only possible stable equilibrium at $\alpha = \alpha^\gamma$ would be the one with $p^e = p^*$.

Combining the above analysis for period 1 with that for period 2, we see that when the (stable) high-safety equilibrium exists, it is unique with all firms charging $p^* = q^*$ in both periods and earning total profits $2\Pi^* - k$ in two periods together. We next characterize the conditions for the existence of the equilibrium. Recall that the high-safety equilibrium does not exist if $\alpha < \alpha^\gamma$. Suppose that $\alpha \geq \alpha^\gamma$. At the proposed equilibrium where all firms choose high safety, if a firm, say firm 1, charges a price leading the consumer to believe that it has high safety, given $\alpha \geq \alpha^{\gamma}$, the firm should have no incentive to deviate to low safety. Thus, if firm 1 indeed deviates to low safety, it would choose a price leading the consumer in period 1 to believe that it has low safety. Then similar to the analysis in the paper, no matter whether firm 1 has positive or zero output in period 1, the consumer in period 2 will hold the correct belief that product 1 has low safety. Given that all the other firm charges $p^*$, the demand for product 1 in period 1 would be $Q_1(p_1, p_2 = p^*; B_1 = D, B_2 = d)$.

Ignoring the constraint $B_1(p_1, p^*) = D$, firm 1's optimal price after deviation in period 1 therefore solves:

$$\max_{p_1}[p_1 - \alpha\theta(D - \delta^*)]Q_1(p_1, p_2 = p^*; B_1 = D, B_2 = d).$$

The optimal deviating price is $\tilde{p}_1 = \alpha\theta(D - \delta^*) + \frac{A-\theta(D-\delta^*)-\phi(\delta^*)}{2} - \frac{1}{2(2-\gamma)}[A - \theta(d - \delta^*) - \phi(\delta^*) - c]$, if and only if $B_1(\tilde{p}_1, p^*) = D$. If $B_1(\tilde{p}_1, p^*) = D$, firm 1's deviating profit in period 1 is

$$\Pi_1^d = \frac{\{\frac{A-\theta(D-\delta^*)-\phi(\delta^*)}{2} - \frac{1}{2(2-\gamma)}[A - \theta(d - \delta^*) - \phi(\delta^*) - c]\}^2}{(1 - \gamma)(1 + \gamma)},$$

which does not depend on $\alpha$.

Accordingly, if $B_1(\tilde{p}_1, p^*) = D$, the difference between firm 1's equilibrium profit and deviation profit in two periods is $\Pi^* - \Pi_1^d + \Delta - k$, which does not depend on $\alpha$. In this scenario, the high-safety equilibrium exists as long as $\Pi^* - \Pi_1^d + \Delta \geq k$.

To summarize, we have derived a sufficient condition for the existence of the stable high-safety equilibrium: $\alpha \geq \alpha^{\gamma}$, and $\Pi^* - \Pi_1^d + \Delta \geq k$:

**Proposition A 3** *There exists a unique equilibrium where all firms produce the high-safety product and charge $p^*$ in both periods if*

$$\alpha \geq \alpha^{\gamma}, \ \textit{and} \ (\Pi^* - \Pi_1^d) + \Delta \geq k.$$

**Optimal Liability**

According to Lemma A4, in the high-safety equilibrium with both firms charging $p^*$, both equilibrium profit and consumer utility do not depend on the liability level $\alpha$. Therefore, $\alpha$ affects social welfare only by changing firms' investment incentives. Notice that $\Pi^* - \Pi_1^d + \Delta$ does not depend on $\alpha$ either. Now suppose that $\Pi^* - \Pi_1^d + \Delta \geq k$. Define the socially optimal liability as $\alpha^*$. Lemma A1 and Proposition A3 imply that whenever $\alpha^* \in (0, 1]$, we have $\alpha^* = \alpha^{\gamma}$. Recall that $\alpha^{\gamma}$ satisfies $\Delta = k - (\alpha^{\gamma}\theta z - c)Q_j(p_j = p_{-j} = p^*; B_j = B_{-j} = d)$. Therefore, whenever $\alpha^* \in (0, 1]$, it satisfies

$$\begin{aligned} k &= Q_j(p_j = p_{-j} = p^*; B_j = B_{-j} = d)[\alpha^*\theta z - c] + \Delta \\ &= \frac{1}{(1+\gamma)(2-\lambda)}[A - c - \theta(d - \delta^*) - \phi(\delta^*)][\alpha^*\theta z - c] \\ &\quad + \frac{1}{(1-\gamma)(1+\gamma)}\{\frac{1-\gamma}{2-\gamma}[A - c - \theta(d - \delta^*) - \phi(\delta^*)]\}^2 \\ &\quad - \frac{1}{(1-\gamma)(1+\gamma)}\{\frac{1-\gamma}{2-\gamma}[A - c - \theta(d - \delta^*) - \phi(\delta^*)] - \frac{2-\gamma^2}{4-\gamma^2}(\theta z - c)\}^2. \end{aligned}$$

It is quite complex to investigate the general relationship between $\alpha^*$ and $\gamma$. Instead, we conduct various numerical examples and find that, whenever $\alpha^* \in (0,1]$, it increases in $\gamma$. For illustration, consider the following numerical example.

**Example A 1** *Consider parameter values such that* $A - c - \theta(d - \delta^*) - \phi(\delta^*) = 5$, $\theta z = 2, c = 1$ *and* $k = 2$. *Then, we find that* $\alpha^* \in (0,1]$ *for any* $\gamma \in [0, 0.8001]$, *and, within this range, the optimal liability* $\alpha^*$ *increases in* $\gamma$ , *as illustrated in the figure below:*

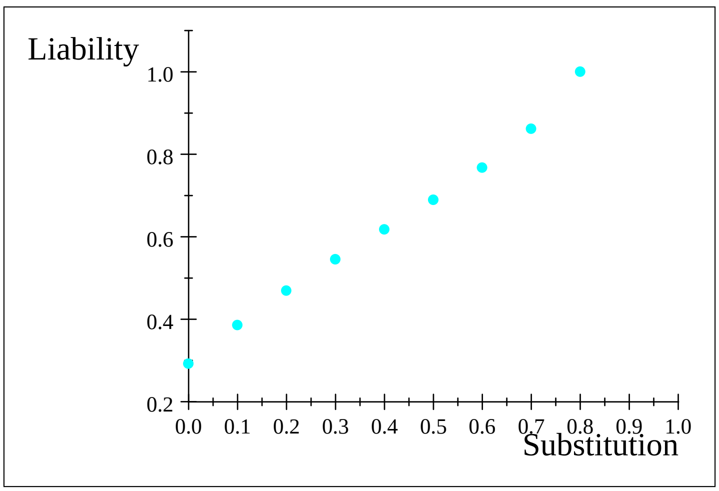


Figure A1: Optimal Liability

The numerical example illustrates that, when competition is more fierce due to the increased product substitution ($\gamma$), the optimal liability should be increased to maintain the firms' investment incentives. This result is similar to that under the spatial model in the paper.

## A3. CONSUMER PRECAUTION CONTINGENT ON PRODUCT SAFETY

In our main model, consumer precaution is assumed to reduce the harm of a faulty product by a certain amount ($\delta$), and the precaution effort is independent of the product safety level in equilibrium. An alternative assumption could be that consumer precaution can reduce the damage by a fraction. Then, as we show in this section, in equilibrium consumer precaution effort will depend on their belief about product safety. Nevertheless, our main results still hold: when competition increases due to less product differentiation, the optimal liability becomes higher; whereas when competition increases due to a larger number of firms, the optimal liability can vary non-monotonically.

Suppose that a consumer can reduce the damage by a fraction of $1 - \eta \in [0, 1)$. This is equivalent to assume that a consumer can reduce the probability of harm to be $\eta\theta$. Note that lower $\eta$ implies less expected damage. Each consumer's precaution cost is $\varphi(\eta)$, which is strictly decreasing and convex. In addition, $\varphi'(0) = -\infty$.

If a consumer uses a high safety product (or believes using a high safety product), she will choose $\eta$ to minimize her expected loss from product malfunction (excluding liability compensation):

$$\max_{\delta} \left\{-(1-\alpha)\theta\eta d - \varphi(\eta)\right\},$$

and the optimal precaution $\eta(d, \alpha)$ satisfies the first order condition:

$$-(1-\alpha)\theta d - \varphi'(\eta(d,\alpha)) = 0.$$

Similarly, if a consumer uses a low safety product, her optimal precaution $\eta(D,\alpha)$ satisfies the first order condition:

$$-(1-\alpha)\theta D - \varphi'(\eta(D,\alpha)) = 0.$$

Given the convexity of $\varphi(\eta)$, we have $\eta(D,\alpha) \leq \eta(d,\alpha)$. That is, consumers take more precaution if they believe that the product has low safety. Also, both $\eta(D,\alpha)$ and $\eta(d,\alpha)$ increase in $\alpha$. That is, when liability is smaller, given product safety, consumers take more precaution which leads to lower $\eta$.

Additionally, based on the above first order conditions and convexity of $\varphi(\eta)$, we have $\frac{\partial\eta(D,\alpha)}{\partial\alpha} > \frac{\partial\eta(d,\alpha)}{\partial\alpha}$, which leads to

$$\frac{\partial\eta(D,\alpha)}{\partial\alpha}D > \frac{\partial\eta(d,\alpha)}{\partial\alpha}d.$$

To simplify notations, define the sum of consumer damage and precaution cost as $\Phi(D,a) \equiv \theta\eta(D,\alpha)D+\varphi(\eta(D,\alpha))$ and $\Phi(d,a) \equiv \theta\eta(d,\alpha)d+\varphi(\eta(d,\alpha))$. Given the first order conditions on consumer precaution and the result $\frac{\partial\eta(D,\alpha)}{\partial\alpha}D > \frac{\partial\eta(d,\alpha)}{\partial\alpha}d$, we can show that $\Phi(D,a) - \Phi(d,a)$ increases in $\alpha$. That is, the difference in expected consumer damage between high safety and low safety products increases in product liability. Notice that, when $\alpha = 1$, consumers expect to receive full compensation and therefore would not take any precaution, that is, $\eta(D,\alpha) = \eta(d,\alpha) = 1$. Correspondingly, $\Phi(D,1) - \Phi(d,1) = \theta(D-d) = \theta z$.

The following lemma summarizes the above analysis on consumer precaution.

**Lemma A 5** *Consumers' optimal precaution depends on their belief about the firms' safety* *investments. The optimal precaution levels, $\eta(D,\alpha)$ and $\eta(d,\alpha)$, increase in $\alpha$; and the difference in expected consumer damage, $\Phi(D,a)-\Phi(d,a)$, increases in $\alpha$ and is not higher than $\theta z$. Given the firms' safety investments and consumer belief, total welfare is higher when product liability $\alpha$ is lower.*

As in the paper, we maintain the assumption $t > \frac{\theta z - c}{2}$ , under which all the firms will have positive outputs both along the equilibrium and off the equilibrium. Given Lemma A5, this assumption also implies that $t > \frac{\Phi(D,a)-\Phi(d,a)-c}{2}$ for any $\alpha \leq 1$.

**Analysis for Period 2**

We focus on the symmetric high safety equilibrium. In period 2, consumers observe each firm's product safety. Suppose that the prices are $q_j$, $j = 1, ..., N$. For any consumer $x_{ij} \in [0,1]$ on $l_{ij}$ , $i \neq j$, and $i, j \in \{1, 2, ..., N\}$, she is indifferent between products $i$ and $j$ if

$$V - x_{ij}t - q_i - [(1-\alpha)\theta\eta(d,\alpha)d + \varphi(\eta(d,\alpha))] = V - (1-x_{ij})t - q_j - [(1-\alpha)\theta\eta(d,\alpha)d + \varphi(\eta(d,\alpha))].$$

Given that all other firms charge $q^*$, the per-period demand for product $j$ is

$$\sum_{i\neq j, i\in\{1,..N\}} \frac{2}{N(N-1)} \max\{\frac{t-q_j+q^*}{2t}, 0\} = \frac{2}{N} \max\{\frac{t-q_j+q^*}{2t}, 0\},$$

provided that $\frac{2}{N} \max\left\{\frac{t-q_j+q^*}{2t}, 0\right\} < 1$.

In period 2, along the equilibrium path, firm $j$ chooses price $q_j$ to maximize its profit:

$$\max_{q_j}[q_j - c - \alpha\theta\eta(d,\alpha)d]\frac{2}{N}\max\left\{\frac{t - q_j + q^*}{2t}, 0\right\},$$

Similar to the analysis in the paper, it is straightforward to show the following

**Lemma A 6** *Suppose that all firms choose high safety. In period 2, there is a unique symmetric equilibrium where each firm sets price* $q^* = t + c + \alpha\theta\eta(d,\alpha)d$ *, sells* $\frac{1}{N}$ *units of output, and earns a profit of* $\Pi^* = \frac{t}{N}$.

If a firm, say firm 1, has deviated to $I = 0$, as long as it has positive output, it will be known as the low-safety firm in period 2. As in the paper, we assume that, if a firm has zero output in period 1, consumers and other firms in period 2 will hold the same belief as consumers in period 1. First, consider the case with all firms having positive outputs in period 1. Consumers purchasing product 1 will take precaution $\eta(D,\alpha)$. Then the demand for product 1 in Period 2 is

$$F_1(q_1, ..., q_N) = \frac{2}{N(N-1)}\sum_{j\in\{2,..N\}}\max\left\{\min\left\{\frac{t - q_1 + q_j - G(\alpha)}{2t}, 1\right\}, 0\right\}.$$

where $G(\alpha) = [(1-\alpha)\theta\eta(D,\alpha)D + \varphi(\eta(D,\alpha)) - (1-\alpha)\theta\eta(d,\alpha)d - \varphi(\eta(d,\alpha))]$.

Firm 1 chooses $q_1$ to maximize its profit in Period 2:

$$\max_{q_1}[q_1 - \alpha\theta\eta(D,\alpha)D]F_1(q_1, ..., q_N).$$

The total demand for any product $j \neq 1$ in Period 2 is

$$\begin{aligned} F_j(q_1, ..., q_N) &= \frac{2\max\left\{\min\left\{\frac{t+q_1-q_j+G(\alpha)}{2t}, 1\right\}, 0\right\}}{N(N-1)} \\ &+ \sum_{i\neq j, i\neq 1, i\in\{1,..N\}} \frac{2}{N(N-1)} \max\left\{\min\left\{\frac{t+q_i-q_j}{2t}, 1\right\}, 0\right\}. \end{aligned}$$

Firm $j \neq 1$ chooses $q_j$ to maximize its profit in Period 2:

$$\max_{q_j} [q_j - c - \alpha\theta\eta(d,\alpha)d]\, F_j(q_1, ..., q_N).$$

Given $t > \frac{\theta z - c}{2}$, it can be verified that $F_1(q_1, ..., q_N) > 0$ and $F_j(q_1, ..., q_N) > 0$. We have the optimal prices for firm 1 and any firm $j \neq 1$ as

$$\begin{aligned} \tilde{q}_1 &= t + \alpha\theta\eta(D,\alpha)D - \frac{N-1}{2N-1}[\Phi(D,a) - \Phi(d,a) - c], \\ \tilde{q}_j &= t + c + \alpha\theta\eta(d,\alpha)d + \frac{1}{2N-1}[\Phi(D,a) - \Phi(d,a) - c] \text{ for any } j \neq 1. \end{aligned}$$

Accordingly, firm 1's deviating profit in period 2 is

$$\Pi_2^d(N,\alpha) = [\tilde{q}_1 - \alpha\theta\eta(D,\alpha)D]F_1(\tilde{q}_1, ..., \tilde{q}_N) = \frac{1}{Nt}\{t - \frac{N-1}{2N-1}[\Phi(D,a) - \Phi(d,a) - c]\}^2.$$

Thus, the reputation loss for the deviating firm in period 2 is

$$\Omega(N,\alpha) \equiv \frac{1}{N}\{t - \frac{[t - \frac{N-1}{2N-1}(\Phi(D,a) - \Phi(d,a) - c)]^2}{t}\}.$$

Similar to the analysis in the paper, it can be verified that $\Omega(N, \alpha)$ decreases in $N$ and increases in $t$. Since Lemma A5 implies that $\Phi(D, a) - \Phi(d, a)$ increases in $\alpha$, $\Omega(N, \alpha)$ increases in $\alpha$. To summarize,

**Lemma A 7** *The reputation loss for a deviating firm, $\Omega(N, \alpha)$, strictly decreases in $N$ and strictly increases in $t$ and $\alpha$.*

**Analysis for Period 1**

In period 1, consumers only observe firms' prices but not product safety. Similar to the analysis in the paper, we assume that consumers will believe a firm to have chosen product safety optimally, given its deviating price and the other firms' equilibrium prices. In particular, denote $p^e$ as the equilibrium price in period 1 in a symmetric high-safety equilibrium, and $B(p_j, p^e) = \{d, D\}$ as consumer belief about product $j$'s safety when firm $j$ charges $p_j$ while all other firms charge the same equilibrium price. Given any price $p_j \neq p^e$ and the prior that all products have high safety, if choosing high safety indeed can generate total profit (in two periods) for firm $j$ no less than choosing low safety, then consumer belief should be $B(p_j, p^e) = d$. In contrast, given $p_j \neq p^e$ and the prior that all products have high safety, if choosing low safety can generate strictly more profit for firm $j$ than choosing high safety, then consumer belief should be $B(p_j, p^e) = D$.

If $p_j \geq p^e + t$, no matter what their belief is, consumers would not buy product $j$ so that firm $j$ has zero output in period 1. Same as in the paper, we assume that, if $p_j \geq p^e + t$, consumers in period 1 hold belief that product $j$ has low safety, and consistently, consumers and other firms in period 2 will hold the same belief that product $j$ has low safety. For any $p_j \leq p^e - t$, firm $j$'s output remains the same and therefore consumer belief satisfies $B(p_j, p^e) = B(p^e - t, p^e)$. For $p_j \in [p^e - t, p^e + t)$, given the prior that all products have high safety, firm $j$'s total profit in two periods by choosing high safety is

$$-k + [p_j - c - \alpha\theta\eta(d,\alpha)d]\frac{t - p_j + p^e}{Nt} + \Pi^*.$$

Similarly, firm $j$'s total profit in two periods by choosing low safety is

$$[p_j - \alpha\theta\eta(d,\alpha)D]\frac{t - p_j + p^e}{Nt} + \Pi_2^d(N,\alpha).$$

Therefore, given $p_j$ and $p^e$, choosing high safety can bring larger profit to firm $j$ if and only if

$$\Omega(N,\alpha) \geq k - (\alpha\theta\eta(d,\alpha)z - c)\frac{t - p_j + p^e}{Nt}. \tag{A4}$$

Notice that the right hand side of condition (A4) is increasing in $p_j$ when $\alpha\theta\eta(d,\alpha)z - c > 0$ while decreasing in $p_j$ when $\alpha\theta\eta(d,\alpha)z - c < 0$. Then, for $p_j \in [p^e - t, p^e + t)$, there exists some unique

$$\widehat{p} \text{ solving } \Omega(N,\alpha) = k - (\alpha\theta\eta(d,\alpha)z - c)\frac{t - \widehat{p} + p^e}{Nt},$$

such that consumer belief takes the following form:

(1) If $\alpha\theta\eta(d,\alpha)z - c = 0$, then $B(p_j, p^e) = d$ if $\Omega(N,\alpha) \geq k$ while $B(p_j, p^e) = D$ if $\Omega(N,\alpha) < k$;

(2) If $\alpha\theta\eta(d,\alpha)z - c < 0$, then $B(p_j, p^e) = d$ if $p_j \geq \widehat{p}$ while $B(p_j, p^e) = D$ if $p_j < \widehat{p}$;

(3) If $\alpha\theta\eta(d,\alpha)z - c > 0$, then $B(p_j, p^e) = d$ if $p_j \leq \widehat{p}$ while $B(p_j, p^e) = D$ if $p_j > \widehat{p}$.

Define $\alpha^N$ such that $\Omega(N,\alpha) = k - \frac{\alpha^N\theta\eta(d,\alpha^N)z - c}{N}$. It is easy to verify that $\widehat{p} \neq p^e$ if $\alpha > \alpha^N$,while $\widehat{p} = p^e$ if $\alpha = \alpha^N$. Similar to the analysis in the paper, it can be shown that high-safety equilibrium does not exist if $\alpha < \alpha^N$. Furthermore, if $\alpha \geq \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c = 0$, we always have $\Omega(N,\alpha) \geq k$ and therefore $B_j(p_j, p^e) = d$.

Now suppose that $\alpha \geq \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c \neq 0$. The above analysis implies that, if firm $j$ charges a price which is in a small neighbourhood of $p^e$, consumers hold belief that product $j$ has high safety. Thus, in any high-safety equilibrium, the optimal price for firm $j$ solves the following problem:

$$\max_{p_j}[p_j - c - \alpha\theta\eta(d,\alpha)d]\frac{t - p_j + p^e}{Nt}.$$

We then have the optimal price for firm $j$ satisfying $p^e = p^* = t + c + \alpha\theta\eta(d,\alpha)d$. Notice that, if firm $j$ chooses a price much higher or lower than $p^*$ so that consumers would believe that it has deviated to low safety, then its profit in period 1 would be lower. In sum, whenever $\alpha \geq \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c \neq 0$, if a high-safety equilibrium exists, it must be unique with all firms charging $p^*$ and earning $\Pi^* - k$ in period 1.

However, when $\alpha = \alpha^N$ and $\alpha\theta\eta(d,\alpha)z - c \neq 0$, without further refinement, we can have a continuum of high-safety equilibria. We restrict our equilibrium concept to "stable equilibrium" as defined in the paper. Thus, when the stable high-safety equilibrium exists, it must be unique with $p^e = p^*$. We next characterize the conditions for the existence of the equilibrium. Recall that the high-safety equilibrium does not exist if $\alpha < \alpha^N$. Suppose that $\alpha \geq \alpha^N$. At the proposed equilibrium where all firms choose high safety, if a firm, say firm 1, charges a price leading consumers to believe that it has high safety, given our earlier analysis, the price must be less than $p^* + t$ and therefore firm 1 has positive output, which reveals its true product safety to consumers and other firms in period 2. Then, as long as $\alpha \geq \alpha^N$, based on our earlier analysis, the firm will have no incentive to deviate to low safety.

Thus, if firm 1 indeed deviates to low safety, it would choose a price leading consumers to believe that it has low safety. Given that all the other firms charge $p^*$, the demand for product 1 in period 1 would be

$$\frac{2}{N} \max \left\{ \frac{t - p_1 + p^* - G(\alpha)}{2t}, 0 \right\},$$

where we recall $G(\alpha) = [(1-\alpha)\theta\eta(D,\alpha)D + \varphi(\eta(D,\alpha)) - (1-\alpha)\theta\eta(d,\alpha)d - \varphi(\eta(d,\alpha))]$.

Similar to the analysis in the paper, the deviating firm's profit in period 2 would be the same no matter whether it has positive or zero output in period 1. Ignoring the constraint $B(p_1, p^*) = D$, firm 1's optimal price after deviation therefore maximizes its period 1 profit:

$$\max_{p_1} \left\{ [p_1 - \alpha\theta\eta(D,\alpha)D] \frac{2}{N} \max \left\{ \frac{t - p_1 + p^* - G(\alpha)}{2t}, 0 \right\} \right\}.$$

Given $t > \frac{\theta z - c}{2}$, the optimal deviating price is $\tilde{p}_1 = t + \alpha\theta\eta(D,\alpha)D - \frac{[\Phi(D,a) - \Phi(d,a)] - c}{2}$, if and only if $B(\tilde{p}_1, p^*) = D$. If $B(\tilde{p}_1, p^*) = D$, firm 1's deviating profit in period 1 is

$$\Pi_1^d = [\tilde{p}_1 - \alpha\theta\eta(D,\alpha)D]\frac{t - \frac{[\Phi(D,a)-\Phi(d,a)]-c}{2}}{Nt} = \frac{(t - \frac{[\Phi(D,a)-\Phi(d,a)]-c}{2})^2}{Nt}.$$

Accordingly, if $B(\tilde{p}_1, p^*) = D$, the difference between firm 1's equilibrium profit and deviation profit in two periods is $\Pi^* - \Pi_1^d + \Omega(N,\alpha) - k$. In this scenario, the high-safety equilibrium exists if $\Pi^* - \Pi_1^d + \Omega(N,\alpha) \geq k$.

To summarize, we have derived a sufficient condition for the existence of the stable high-safety equilibrium: $\alpha \geq \alpha^N$, and $\Pi^* - \Pi_1^d + \Omega(N,\alpha) \geq k$. Similar to the analysis in the paper, it can be shown that this condition is also necessary. $\alpha \geq \alpha^N$ can be re-written as $\Omega(N,\alpha) + \frac{\alpha\theta\eta(d,\alpha)z-c}{N} \geq k$, and $\Pi^* - \Pi_1^d + \Omega(N,\alpha) \geq k$ can be re-written as $\Omega(N,\alpha) + \frac{t}{N} - \frac{(t-\frac{[\Phi(D,a)-\Phi(d,a)]-c}{2})^2}{Nt} \geq k$. Combing these conditions, we have

**Proposition A 4** *There exists a unique equilibrium where all firms produce the high-safety product and charge* $p^* = t + c + \alpha\theta\eta(d,\alpha)d$ *in both periods if and only if*

$$\frac{1}{N}\min\{\alpha\theta\eta(d,\alpha)z - c, t - \frac{(t - \frac{[\Phi(D,a)-\Phi(d,a)]-c}{2})^2}{t}\} + \Omega(N,\alpha) \geq k. \tag{A5}$$

**Optimal Liability**

Lemma A7 shows that $\Omega(N,\alpha)$ increases in $\alpha$. In addition, given that $\eta(d,\alpha)$ and $\Phi(D,a)-\Phi(d,a)$ increase in $\alpha$, it is easy to verify that the left-hand side of condition (A5) increases in $\alpha$. Then given Lemma A5, it can be shown that, whenever the optimal liability $\alpha^* > 0$, it satisfies the following condition:

$$\Omega(N,\alpha^*) = k - \frac{1}{N}\min\{\alpha^*\theta\eta(d,\alpha^*)z - c, t - \frac{(t - \frac{[\Phi(D,\alpha^*)-\Phi(d,\alpha^*)]-c}{2})^2}{t}\}, \tag{A6}$$

where the term on the right-hand side is a deviating firm's gain in period 1, which decreases in $\alpha^*$ and $t$; while $\Omega(N,\alpha^*)$ is the reputation loss, which increases in $\alpha^*$ and $t$. Then, for the range of $t$ under which $\alpha^* > 0$, when $t$ decreases, to maintain condition (A6) , the optimal liability $\alpha^*$ is higher.

**Proposition A 5** *Suppose that $\alpha^* > 0$ for $t \in [t_L, t_H]$. Then for any $t \in [t_L, t_H]$, the optimal liability $\alpha^*$ decreases in $t$.*

Finally, $\Omega(N,\alpha^*)$ decreases in $N$. Furthermore, the right hand side of condition (A6) decreases in $N$ if $\min\{\alpha^*\theta\eta(d,\alpha^*)z - c, t - (t - \frac{[\Phi(D,\alpha^*)-\Phi(d,\alpha^*)]-c}{2})^2\} < 0$, while increases in $N$ if otherwise. Notice that these effects are similar to those in the paper. Thus, similar to the analysis in the paper, the non-monotonic relationship between $\alpha^*$ and $N$ can hold.

**Proposition A 6** *Suppose that $\alpha^* > 0$ for $N \in [N_1, N_2]$, with $2 \leq N_1 < N_2 \leq \overline{N}$. There exists some $\widehat{N} \in [N_1, N_2]$ such that, for any $N \in [N_1, N_2]$, the optimal liability $\alpha^*$ decreases in $N$ for $N < \widehat{N}$ and increases in $N$ for $N > \widehat{N}$.*

To summarize, under this alternative model where consumer precaution depends on their belief about product safety, our main results about the relationship between competition and the optimal liability are still robust.

## A4. IMPERFECT OBSERVATION OF PAST DAMAGE

Our main model assumes that consumers in period 2 can observe what happened in period 1. Hence, from damages (and output levels) occurred in period 1, consumers can detect any firm who has deviated to selling a low safety product. While this perfect observability is clearly a strong assumption, which greatly simplifies our analysis, in this section, we show that our results can continue to hold if our model is extended to allow imperfect observability. In particular, assume that, with probability $r \in (0,1]$, all consumers and the firms in period 2 observe what happened in period 1; with probability $1-r$, no consumer (and no other firm) observes past damage or output levels. In reality, such imperfect and symmetric observation could arise due to the costs or imperfect implementation of disclosure regarding market information and consumer damage.

We focus on the stable high-safety equilibrium. Notice that the analysis of the equilibrium price and profit is the same as under the basic model in the paper. In particular, in period 2, with probability $r$, consumers and the firms observe past damage and output levels, and therefore have the correct belief about product safety. Under this scenario, the analysis of period 2 is similar to that under the basic model. If a firm, say firm 1, deviates to low safety, its expected reputation loss is

$$r\Delta(N) \equiv \frac{r}{N}\left[t - \frac{(t - t_2(N))^2}{t}\right].$$

In period 2, with probability $1-r$, consumers and the firms do not observe past damage or past output levels. Under this scenario, the analysis of period 2 is the same as that of period 1 under the basic model. Define $\widetilde{\alpha}^N$ such that $r\Delta(N) = k - (2-r)\frac{\widetilde{\alpha}^N\theta z - c}{N}$. Then similar to the analysis in the paper, it can be shown that the stable high-safety equilibrium is unique with all firms charging $p^*$ in both periods, and this equilibrium exists if and only if

$$\alpha \geq \widetilde{\alpha}^N \text{ and } (2-r)[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + r\Delta(N) \geq k.$$

Therefore, the socially optimal liability $\alpha^*$ sustaining the high-safety equilibrium, whenever positive, equals $\widetilde{\alpha}^N$ and satisfies

$$r\Delta(N) = k - (2-r)\frac{\alpha^*\theta z - c}{N}.$$

The above characterization of the optimal liability is similar to that under the basic model. Then similar to the proof of Proposition 3 in the paper, it can be shown that $\alpha^*$ may change non-monotonically when $N$ increases. Additionally, following the same proof of Proposition 2 in the paper, it can be shown that, within any range of $t$ such that $\alpha^* > 0$, the optimal liability $\alpha^*$ strictly decreases in $t$. To conclude, the main results in the paper are robust under the extension with the above imperfect and symmetric observation of past damage.

As a remark, another potential extension is to consider imperfect and asymmetric observability, where in period 2 all the firms and a fraction of consumers observe what happened in period 1, while the rest consumers do not observe past damage or past output levels. In this scenario, there exists asymmetric information about product safety between the firms and the uninformed consumers in period 2. If one firm deviates to low safety while all other firms sell the high-safety product, then in the sub-game of period 2, there can be multiple separating pricing equilibria and even pooling equilibria, depending on the assumptions on consumer belief in this off-equilibrium path. The full-fledged analysis would become more complex and beyond the scope of this paper.

## A5. SMALL CONSUMER HETEROGENEITY

In the main model, we assume that consumer heterogeneity is large enough ($t > \frac{\theta z - c}{2}$) so that firms always have positive outputs both along the equilibrium and in the off-equilibrium path. In this section, we show that our main results concerning the relationship between the optimal liability and competition intensity stay robust, even if consumer heterogeneity is small. In particular, assume that $t \leq \frac{\theta z - c}{2}$. Again, we focus on the stable high-safety equilibrium. If a firm has positive output in period 1, consumers and other firms in period 2 will observe its true product safety; if a firm has zero output in period 1, then consumers and other firms in period 2 will hold the same belief as consumers in period 1. Then the analysis of the equilibrium in period 2 is the same as in the paper: a firm's equilibrium profit in period 2 is $\frac{t}{N}$.

If a firm, say firm 1, has deviated to low safety, as long as it has positive output in period 1, it will be known as the low safety firm in period 2. Similar to the analysis in the paper, firm 1 then has positive output in period 2 if and only if $t > t_2 = t_2(N) = \frac{N-1}{2N-1}(\theta z - c)$. Given the assumption $t \leq \frac{\theta z - c}{2}$, there exists some unique $\widetilde{N} \geq 2$ such that, for $N < \widetilde{N}$, $t > t_2(N)$, and for $N \geq \widetilde{N}$, $t \leq t_2(N)$. Notice that, if $t \leq \frac{\theta z - c}{3}$, $\widetilde{N}$ equals 2. Therefore, when $N < \widetilde{N}$, the deviating firm's profit in period 2 would be $\frac{(t-t_2)^2}{Nt}$; when $N \geq \widetilde{N}$, the deviating firm's output and profit in period 2 would be zero. Then the firm's reputation loss can be defined as

$$\Gamma(N) \equiv \frac{t}{N} - \lambda_{N<\widetilde{N}} \frac{(t - t_2(N))^2}{Nt},$$

where $\lambda_{N<\widetilde{N}}$ is an indicator function that equals 1 if $N < \widetilde{N}$ and 0 otherwise. Similar to the analysis in the paper, the reputation loss $\Gamma(N)$ can be shown to be decreasing in $N$ and increasing in $t$.

The construction of consumer belief in period 1, as well as the analysis of the game in period 1, is similar to that in the paper. The only difference is that, given $t \leq \frac{\theta z - c}{2}$, if a firm, say firm 1, deviates to low safety and charges a price in period 1 such that $B(p_1, p^*) = D$, its optimal output would be zero. To see this, notice that, given $B(p_1, p^*) = D$, the demand for product 1 in period 1 is

$$\frac{2}{N} \max\{\frac{t - p_1 + p^* - (1-\alpha)\theta z}{2t}, 0\},$$

provided $\frac{t-p_1+p^*-(1-\alpha)\theta z}{2t} < 1$.

Given consumer belief in period 1 as $B(p_1, p^*) = D$, no matter whether firm 1 has positive or zero output in period 1, consumers and other firms in period 2 will have the correct belief that product 1 has low safety. Firm 1's optimal price after deviation in period 1 therefore solves:

$$\max_{p_1}[p_1 - \alpha\theta(D-\delta^*)]\frac{2}{N}\max\{\frac{t-p_1+p^*-(1-\alpha)\theta z}{2t},0\}.$$

The optimal deviating price is $\tilde{p}_1 = t + \alpha\theta(D-\delta^*) - \frac{\theta z - c}{2}$. Then given $t \leq \frac{\theta z - c}{2}$, firm 1's profit margin is non-positive and therefore it prefers to have zero output in period 1.

Accordingly, if $t \leq \frac{\theta z - c}{2}$ and $B(\tilde{p}_1, p^*) = D$, the difference between firm 1's equilibrium profit and deviation profit in two periods is

$$(\frac{2t}{N} - k) - [0 + \lambda_{N<\tilde{N}}\frac{(t - t_2(N))^2}{Nt}] = \frac{t}{N} + \Gamma(N) - k.$$

Let $\alpha^N$ be such that $\Gamma(N) = k - \frac{\alpha^N\theta z - c}{N}$. Then similar to the analysis in the paper, it can be shown that the high-safety equilibrium with all firms charging $p^*$ in both periods exists if and only if

$$\alpha \geq \alpha^N \text{ and } \frac{t}{N} + \Gamma(N) \geq k.$$

Thus, whenever the optimal liability $\alpha^*$ sustaining the high-safety equilibrium is positive, it equals $\alpha^N$ and satisfies

$$\Gamma(N) = k - \frac{\alpha^*\theta z - c}{N}.$$

Notice that the characterization of $\alpha^*$ is similar to that in the paper, except that the reputation loss becomes $\Gamma(N)$, which still increases in $t$ and decreases in $N$. Accordingly, our main results still hold with $t \leq \frac{\theta z - c}{2}$: when competition increases due to less product differentiation, the optimal liability becomes higher; whereas when competition increases due to a larger number of firms, the optimal liability can vary non-monotonically.