Competition, Product Safety, and Product Liability[1]

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Abstract. A firm's incentive to invest in product safety is affected by both market environment and product liability. We investigate the relationship between competition and product liability in a spatial model of oligopoly, where reputation provides a market incentive for safety investment and higher liability may distort consumers' incentive for product care. We find that partial liability, together with reputation concerns, can motivate firms to make safety investment. Increased competition due to less product differentiation diminishes a firm's gain from maintaining reputation and *raises* the socially desired product liability. On the other hand, an increase in the number of competitors reduces the benefit from maintaining reputation, but has a non-monotonic effect on the potential gain from cutting back safety investment; consequently, the optimal liability may vary non-monotonically with the number of competitors. In general, therefore, the relationship between competition and product liability is subtle, depending on how competition is measured.

1. INTRODUCTION

Market competition and product liability are two major mechanisms that affect firms' incentives to increase product safety and prevent product harm to consumers. Extensive studies in law and economics have examined the effects of product liability and derived optimal liability under a given market structure. However, there has been little formal analysis on how competition and product liability may interact to incentivize firms.[4] This is rather surprising, given the importance of product safety in many consumer markets. What is the relationship between competition and the socially desired liability level? Are competition and product liability substitutes or complements in increasing product safety and social welfare? This paper provides an economic analysis that aims to shed light on these questions.

We consider a two-period spatial model with $N \geq 2$ firms selling differentiated products to heterogeneous consumers. The products may malfunction and cause consumer harm with some probability. At the beginning of the first period, each firm can decide whether to invest in product safety. Investment leads to a safer product that causes less damage when it fails. Consumers cannot observe firms' safety investments, but in the second period they can observe the damages to the harmed consumers in the first period and update their beliefs about product safety. That is, there are reputation concerns for firms. If a firm's product causes consumer harm, it may bear product liability and compensate consumers.

In particular, under partial liability the firm is required to compensate only a proportion of consumer loss, whereas under full liability the firm is liable for the full damage. While liability can motivate firms to invest in product safety, it is not without undesirable incentive consequences due to the presence of two-sided moral hazard: After purchase, consumers can take precaution to reduce the potential harm from product failure; and higher product liability lowers the consumers' precaution efforts.

We are interested in situations where investment in product safety is socially desirable. The choice of product liability level is thus concerned with how to sustain the equilibrium where firms make safety investment. If a firm chooses a low-safety product, it faces high (expected) liability costs and reputation loss. When reputation loss is high enough, the firm will make the safety investment even without product liability, in which case the socially optimal liability is zero in order to encourage consumer precaution. However, if reputation concern is not sufficient, then product liability needs to be increased in order to sustain the firm's investment incentive. For a given number of competitors in the market, less horizontal product differentiation reduces equilibrium price, which diminishes the reputation loss in period 2 from offering a low-safety product. Consequently, the socially optimal liability level is larger when competition becomes tougher due to reduced product differentiation. In this sense, competition and product liability are complements in improving product safety and social welfare.

However, the socially optimal liability level may vary non-monotonically with another measure of competition intensity: the number of competitors. If a firm deviates from a high-safety to a low-safety product, its gain in period 1 depends on the fixed investment cost saving and the change in the effective marginal cost, which is the extra liability cost minus variable production cost. A firm would sell a high-safety product only when the reputation loss is larger than the deviation gain in period 1. In our model, by reducing equilibrium output from each firm, an increase in the number of competitors always reduces the reputation loss from deviation, but has a non-monotonic impact on the deviation gain in period 1, depending on the change in the effective marginal cost. As a result, the optimal liability may vary non-monotonically with the number of competitors, possibly first decreasing and then increasing.

Thus, in general, the relationship between competition and product liability is subtle, depending on the measure of competition intensity. While they can often be complements, the relationship may also be non-monotonic when competition is measured by the number of competitors in the market. Our results can shed light on the mixed empirical evidence concerning the effects of competition on firms' investment incentives for product safety. For example, a 2008 survey among product development managers in the US revealed that companies were more likely to reduce safety investment and to speed up new product introductions, possibly with lower safety, when facing more competition (Samra et al., 2008). In the automobile industry, when more companies entered the market of SUVs, many products had low quality and later caused substantial consumer harm (Los Angeles Times, March 14, 2010). However, there have also been empirical studies showing that competition can increase product quality, though most of the studies do not focus on safety issues.[5] Furthermore, there is empirical evidence suggesting that product liability can have non-monotonic effects on firms' innovation incentives (e.g., Viscusi and Moore, 1993).

Our paper contributes to the literature on product liability. Studies with a single firm analyze, for example, the effects of product liability on a firm's precaution to ensure product safety (Simon, 1981), or on both before-sales and after-sales precaution (Ben-Shahar, 1998; Chen and Hua, 2012), or on its incentive to disclose quality information through price and other devices (Daughety and Reinganum, 1995, 2008a).[6] Studies with competition include Epple and Raviv (1978), Polinsky and Rogerson (1983), Cooper and Ross (1984), and Daughety and Reinganum (2006, 2008b, 2014). Polinsky and Shavell (2010) argue that market mechanisms and product liability are substitutes as they both can increase product safety. Ganuza et al. (2016) study the roles of liability and reputation in a repeated game setting with one firm and one consumer. They show that the reputation mechanism can motivate the firm to take effort that reduces the consumer's expected damage, but it causes social loss when the consumer does not buy in the punishment periods. Having liability can reduce such social loss.[7] Our study differs from the literature by providing a formal analysis on the relationship between product liability and competition, and we find that product liability and competition can be either complements or substitutes under alternative measures of competition.

Our paper is also related to the literature in industrial organization, where market reputation can be an effective mechanism to improve product quality (Shapiro, 1983; Allen, 1984; Bagwell and Riordan, 1991; Kranton, 2003), and where market competition may have either positive or negative impacts on product quality (Riordan, 1986; Shaked and Sutton, 1987; Horner, 2002; Dana and Fong, 2011). We depart from the literature by focusing on safety investments and product liability.

The rest of the paper is organized as follows. Section 2 presents our model and derives consumers' precaution effort in equilibrium. Section 3 analyzes firms' investment incentives and how changes in competition, measured alternatively by product differentiation and the number of competitors, affect the socially optimal liability level. Section 4 discusses the robustness of our results and some alternative policies. Section 5 concludes. Proofs of the main results are gathered in the appendix. An online appendix contains the proofs for Lemma 3 and Lemma 5, as well as detailed analysis for the discussion in Section 4 under the alternative modelling assumptions.[8]

2. THE MODEL

A market has $N \geq 2$ firms and a unit mass of consumers. Consumers are uniformly distributed on a network of $\frac{N(N-1)}{2}$ arcs of length 1, and the density of consumers on each arc is thus $\frac{2}{N(N-1)}$. Each firm is uniquely located at one end of each of $N-1$ arcs. In the static form of the model, firms choose prices simultaneously, with firm i competing with every other firm j on a separate arc l_{ij}, for $j \neq i$ and $i, j = 1, ..., N$. Each consumer, who values the product at V and demands at most one unit, must travel to a firm in order to make a purchase, with unit transportation cost $t > 0$. A consumer on l_{ij} is uniquely denoted by $x_{ij} \in [0, 1]$, whose distance is x_{ij} from firm i and $1 - x_{ij}$ from firm j. Consumer x_{ij} will purchase the product if her net surplus from the product — V minus price and transportation cost — is non-negative, and she patronizes the firm with the highest net surplus between the two firms at the two ends of l_{ij} to which she belongs, i and j. Adapted from Chen and Riordan's (2007) spokes model, this model provides a tractable formulation of oligopoly price competition that extends the Hotelling analysis to any number of firms with non-localized competition, where effectively each firm competes with every firm else for different segments of consumers.[9] Notice that this formulation reduces to the standard Hotelling model when $N = 2$. Figure 1 illustrates the model in the cases of $N = 3$ and $N = 4$. Notice also that when another firm is added to an N-firm model, there are N more l_{ij} arcs. Since the overall mass of consumers is held fixed, they are reallocated across the arcs. In Figure 1, when $N = 3$, there are 3 arcs and 1/3 of the consumers are on each arc; if another firm is added, there are now $N + 1 = 4$ firms and 6 arcs, with each arc containing 1/6 of the consumers.

<<COMP: Place Figure 1 about here>>

The static model described above is then embedded into a simple two-period dynamic game with safety investment and product liability. Specifically, each firm's product may cause consumer harm with probability θ. At the beginning of Period 1, a firm can choose to invest $k > 0$, which enables it to produce a high-safety product in both periods at marginal cost $c \geq 0$. Without the investment, the product will have low safety and zero marginal cost. After purchasing a product, a consumer can take precaution effort. Without such effort, if a consumer is harmed, her damage is d from a high-safety product and $D > d$ from a low-safety product.[10] We define $z \equiv D - d$, and assume $c < \theta z$. Then, when the fixed cost of safety investment is sufficiently small, it is efficient for firms to produce and sell the high-safety product.

With precaution effort, a consumer can reduce the damage by $\delta \in [0, d)$.[11] Each consumer's precaution cost is $\phi(\delta)$, which is strictly increasing and convex, with $\phi(0) = 0$, $\phi'(0) = 0$, and $\phi'(d) > \theta$. With consumer precaution, the expected damage level from a high-safety product is $\theta(d - \delta)$, and the expected damage level from a low-safety product is $\theta(D - \delta)$.

If a consumer is harmed, the firm is required to compensate the consumer α fraction of the damage according to its product liability. The firm bears "partial liability" if $\alpha < 1$, "full liability" if $\alpha = 1$ and punitive damage compensation if $\alpha > 1$. For simplicity, we focus on scenarios with $\alpha \leq 1$.[12] Liability α can be interpreted in various ways. For example, under the strict liability rule, α is the share of consumer damage to be born by the firm. Alternatively, α can be interpreted as the probability that court finds the firm liable (and when found liable, the firm pays compensation equal to consumer damage), assuming that court cannot observe or infer the firm's investment decisions or consumers' precaution levels.

In neither period can consumers directly observe the firms' investments or product safety. In period 2, however, firms and consumers observe the damage levels suffered by harmed consumers in period 1. Consequently, they learn about the safety level of each product. In particular, if product j causes damage D (or d) in period 1, then consumers and other firms in period 2 will believe that product j has low safety (or high safety). We denote firm j's total profit in two periods as $\Pi_j \left(I_j, I_{-j} | \, B\right)$, where (I_j, I_{-j}) is a vector of investments by firm j and the $N-1$ other firms, and B is consumers' belief in period 1 about product safety. In our simple setting, as long as all firms have positive outputs in period 1, consumers in period 2 will always learn the true product quality, because with a continuum of consumers, fraction θ of consumers will be harmed in period 1, and the damage levels suffered by them, which reveal product safety levels, are observed by all consumers at the beginning of period 2. If a firm has zero output in period 1 so that information about its past damage is not available, we assume that consumers and other firms in period 2 hold the same belief about that firm as consumers in period 1.[13] Hence beliefs by consumers in period 2 are not shown explicitly in $\Pi_j(I_j, I_{-j}| B)$. In period 1, consumer beliefs are consistent with firms' equilibrium safety choices, and we discuss this further in the next section.

To summarize, the timing of the model is as follows:

- Period 1:
  - Stage 1: Each firm independently decides whether to invest in product safety, $I_j = 0$ or k, $j = 1, 2, ..., N$. Each firm's product safety choice is its private information.
  - Stage 2: Firms simultaneously and independently post their prices p_j, $j = 1, 2..., N$. Consumers, observing the prices (but not the safety choices), make possible purchases.
  - Stage 3: Each consumer chooses her precaution effort δ after purchase.
  - State 4: If any consumer is harmed by a firm's product, the firm bears liability $\alpha \leq 1$.
- Period 2: Consumers observe the damage levels to the harmed consumers.[14] Stages 2-4 in period 1 are then repeated, with firm $j's$ price in period 2 denoted by q_j.

Throughout the paper, we make the following assumption to ensure the full coverage of the market:

A1: $V > \frac{3}{2}t + c + [\theta D + \phi(d)]$.

Before analyzing firm strategy, we first examine consumer precaution effort and the efficient safety investment. If all N firms make the safety investment and all consumers purchase, in two periods the total costs for producing the high-safety product is $Nk + 2c$, with social benefit $2\theta(D - d) = 2\theta z$. Hence, without consumer precaution, it is efficient for all the firms to invest in safety if and only if $2\theta z \geq Nk + 2c$.

Regardless of whether the purchased product has high or low safety, each consumer will choose δ to minimize her expected loss (excluding liability compensation) from product malfunction:

$$\max_{\delta} \left\{(1-\alpha)\theta\delta - \phi(\delta)\right\},$$

and the optimal $\delta^* \equiv \delta(\alpha)$ satisfies the first order condition:

$$(1-\alpha)\theta - \phi'(\delta^*) = 0.$$

Since $\phi(\delta)$ is convex and $\phi'(d) > \theta$, we have $\delta(\alpha) < d$ for any α, and $\delta(\alpha)$ decreases in α. Intuitively, when product liability (α) is larger, consumers expect to receive more compensation from the firm if they are harmed, which reduces their incentive to take precaution. However, efficiency requires that every consumer takes precaution to maximize $\theta\delta - \phi(\delta)$. Therefore, as the result below states, given the firms' investment decisions, the efficiency of consumer precaution effort increases, or total welfare improves, when liability is smaller.[15]

Lemma 1 *Given the firms' safety investments, consumers' precaution effort $\delta(\alpha)$ and total welfare are higher when product liability α is lower.*

Lemma 1 will play an important role in our analysis to follow, where firms' safety investment incentives will be shown to increase in product liability. The incentive conflict of product liability for consumers and firms will lead to some unique (possibly interior) liability that maximizes total welfare.

We will be interested in situations where $2\theta z - Nk - 2c > \max\{\theta\delta - \phi(\delta)\}$, so that investment in high safety is always efficient. For this purpose our analysis will further assume

A2: $N \leq \bar{N}$, with $\bar{N} \equiv \frac{2\theta z - 2c - \max\{\theta\delta - \phi(\delta)\}}{k} > 2$.

3. COMPETITION AND PRODUCT LIABILITY

In this section, we derive the firms' equilibrium strategies and then address two questions. First, given the number of competitors, how will changes in the degree of horizontal product differentiation, a measure of competition intensity, impact the socially optimal liability level? Second, how will the change in market structure (i.e., the number of competitors) affect the optimal liability? In addressing these questions, we also investigate whether market competition and product liability are complements or substitutes in increasing product safety and welfare.

3.1 HIGH-SAFETY EQUILIBRIUM

We focus on the symmetric equilibrium where all N firms invest in product safety (i.e., choose high safety) and charge the same price. To derive the equilibrium, we first characterize the firms' pricing decisions and profits in period 2, and then discuss consumer belief in period 1 and the firms' strategies. For simplicity, we maintain the following assumption in the remaining analysis:

A3: $t > t_1 \equiv \frac{(\theta z - c)}{2}$.[16]

In the symmetric high-safety equilibrium, each firm sells positive output in period 1. Then in period 2, consumers observe each firm's product safety, because with a continuum of consumers, a positive portion of consumers in period 1 have experienced product malfunction from each firm and the damage level reveals each firm's product safety. Along the equilibrium path in period 2, for any consumer $x_{ij} \in [0,1]$ on the arc l_{ij} , $i \neq j$, and $i, j \in \{1, 2, ..., N\}$, she is indifferent between products i and j if

$$V - x_{ij}t - q_i - [(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)] = V - (1-x_{ij})t - q_j - [(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)], \quad (1)$$

where $[(1-\alpha)\theta(d-\delta^*) + \phi(\delta^*)]$, which we shall call a consumer's expected damage, consists of her expected loss when a product malfunctions and her cost of exerting precaution effort. Given the same product safety for both firms, a consumer's expected damages from the two products cancel each other in (1). Hence, given that all other firms charge price q^*, the per-period demand for product j is

$$\sum_{i \neq j, i \in \{1,..N\}} \frac{2}{N(N-1)} \max\{\frac{t - q_j + q^*}{2t}, 0\} = \frac{2}{N} \max\left\{\frac{t - q_j + q^*}{2t}, 0\right\},$$

provided that $\frac{2}{N} \max\left\{\frac{t-q_j+q^*}{2t}, 0\right\} < 1$.

At the high-safety equilibrium, firm j chooses price q_j to maximize its profit in period 2:

$$\max_{q_j} [q_j - c - \alpha\theta(d - \delta^*)] \frac{2}{N} \max\left\{\frac{t - q_j + q^*}{2t}, 0\right\},$$

where $\alpha\theta(d - \delta^*)$ is the firm's expected liability cost per unit of sales. It is straightforward to establish the following:

Lemma 2 *Suppose that all firms choose high safety, i.e., $I = k$. Then, in period 2 there is a unique symmetric equilibrium where each firm sets price $q^* = t + c + \alpha\theta(d - \delta^*)$, sells $\frac{1}{N}$ units of output, and earns a profit of $\frac{t}{N}$.*

Each firm's effective marginal cost in each period is $\hat{c} \equiv [c + \alpha\theta(d - \delta^*)]$, and the equilibrium price in period 2 is $t + \hat{c}$, same as in the standard Hotelling model. Intuitively, each firm's profit decreases when there are more competitors or when there is less product differentiation. Notice that liability does not affect the firms' profits on the equilibrium path, because at the symmetric equilibrium where all firms have the same product safety, consumers face the same expected damage from all firms, so that the liability level merely shifts the equilibrium price (q^*) without affecting the equilibrium markup ($q^* - \hat{c}$).

Suppose that one firm, say firm 1, has deviated to $I = 0$. As long as firm 1 still has positive output in period 1, it will be known as the low-safety firm in period 2. If any firm has zero output in period 1, to be consistent, we assume that consumers and other firms in period 2 hold the same belief about this firm's product safety as consumers in period 1. We shall first analyze the case with all firms having positive outputs. After we characterize the consumer belief in period 1, we then discuss the other case with some firm having zero output.

In the first case, where all firms have positive outputs, suppose that the prices are q_j, $j = 1, ..., N$ following firm 1's deviation. Denote the demand for product 1 in period 2 as $F_1(q_1, ..., q_N)$, and demand for any other product $j \neq 1$ as $F_j(q_1, ..., q_N)$. Notice that, for any firm $j \neq 1$, it competes for two types of consumers: consumers located on l_{1j}, and consumers located on l_{ij}, for any $i \neq j$ and $i \neq 1$. Firm 1 chooses q_1 to maximize its profit in Period 2:

$$\max_{q_1} [q_1 - \alpha\theta(D - \delta^*)] F_1(q_1, ..., q_N).$$

And firm $j \neq 1$ chooses q_j to maximize its profit in Period 2:

$$\max_{q_j} [q_j - c - \alpha\theta(d - \delta^*)] F_j(q_1, ..., q_N).$$

As we show in the online appendix, with firms choosing Nash equilibrium prices in the subgame of period 2, $F_1(q_1, ..., q_N) > 0$ if $t > \frac{N-1}{2N-1}(\theta z - c)$, which holds for any N given assumption A3 ($t > \frac{(\theta z - c)}{2}$).[17] We then have:

Lemma 3 *Suppose that firm 1 has low safety while the other firms have high safety, and that all firms have positive outputs in period 1. Then, firm 1's profit in period 2, with firms choosing Nash equilibrium prices in this subgame, is $\frac{(t-t_2)^2}{Nt}$, where $t_2 \equiv t_2(N) = \frac{N-1}{2N-1}(\theta z - c)$.*

From Lemma 3, liability does not affect profits in period 2 even when firm 1 has low safety and the other firms have high safety. To see the intuition behind this, consider competition between firm 1 and firm $j \neq 1$ in period 2. On one hand, firm 1 and firm j face different expected liability costs ($\alpha\theta z$), so that they may charge different prices in the subgame. On the other hand, in period 2, consumers have the correct belief about the difference in safety between firm 1 and firm j, and correspondingly the difference in their expected loss ($(1-\alpha)\theta z$), which affects demand levels for the two firms' products. By affecting differences both in firms' liability costs and in consumers' expected losses, liability level (α) only impacts firms' equilibrium prices in this subgame but does not influence their markups or output levels.[18]

From Lemmas 2 and 3, a deviating firm's "reputation loss" in period 2 can be defined as

$$\Delta(N) \equiv \frac{1}{N}\left[t - \frac{(t - t_2(N))^2}{t}\right].$$

Now consider the game in period 1. An important issue for our analysis is to specify consumer belief about product safety in period 1, when consumers only observe firms' prices. While our equilibrium concept, perfect Bayesian equilibrium, requires that consumer beliefs must be consistent with firm strategies along the equilibrium path, it does not constrain off-equilibrium beliefs. Thus, there are potentially different off-equilibrium beliefs that could support a symmetric equilibrium where all firms choose high safety and also the same equilibrium price in period 1. For example, one possibility is that in period 1, when seeing an off-equilibrium price from a firm, consumers continue to believe that the firm has produced a high-safety product. Such an off-equilibrium belief imposes the least punishment on a deviating firm, and if no firm could profitably deviate under such a belief, one might think that the equilibrium is robust in some sense.[19] However, it could be more natural to allow consumers to hold “reasonable” beliefs that interpret off-equilibrium prices as "signals" of product safety, even though our model is not a standard signaling game. For this purpose, we shall adopt the approach used by Dana (2001) in specifying consumer beliefs off the equilibrium path.

In a framework where consumers only observe prices but not firms' capacity choices, Dana (2001) focuses on consumer belief that a firm has chosen capacity optimally, given its deviating price and the other firms' equilibrium prices.[20] Similarly, we assume that consumers will believe the firm to have chosen product safety optimally, given its deviating price and the other firms' equilibrium prices. Specifically, denote p^e as the equilibrium price in period 1 in a symmetric high-safety equilibrium, and $B(p_j, p^e)$ as the consumer belief about product j's safety when firm j charges p_j while all other firms charge p^e. If consumers believe that product j has high safety, then $B(p_j, p^e) = d$; otherwise, $B(p_j, p^e) = D$. Given any price $p_j \neq p^e$ and the prior that all products have high safety, if choosing high safety can indeed bring firm j a total profit (in two periods) no less than that from choosing low safety, then consumer belief should be $B(p_j, p^e) = d$. On the other hand, given $p_j \neq p^e$ and the prior that all products have high safety, if choosing low safety can generate strictly higher profit for firm j than choosing high safety, then consumer belief should be $B(p_j, p^e) = D$.[21] Notice that, given the prior that all products have high safety, firm j has positive output if $p_j < p^e + t$. For any $p_j \leq p^e - t$, firm j has the same output and therefore consumer belief satisfies $B(p_j, p^e) = B(p^e - t, p^e)$. For $p_j \in [p^e - t, p^e + t)$, we show in the appendix that there exists some unique

$$\widehat{p} \text{ solving } \Delta(N) = k - (\alpha\theta z - c)\frac{t - \widehat{p} + p^e}{Nt}, \tag{2}$$

such that consumer belief takes the following form:

(1) If $\alpha\theta z - c = 0$, then $B(p_j, p^e) = d$ if $\Delta(N) \geq k$ while $B(p_j, p^e) = D$ if $\Delta(N) < k$;

(2) If $\alpha\theta z - c < 0$, then $B(p_j, p^e) = d$ if $p_j \geq \widehat{p}$ while $B(p_j, p^e) = D$ if $p_j < \widehat{p}$;

(3) If $\alpha\theta z - c > 0$, then $B(p_j, p^e) = d$ if $p_j \leq \widehat{p}$ while $B(p_j, p^e) = D$ if $p_j > \widehat{p}$.

This set of consumer beliefs has an intuitive interpretation. When deviation to low safety does not change firm j's effective marginal cost ($\alpha\theta z - c = 0$), consumers believe that product j has high safety if the reputation loss $\Delta(N)$ is larger than the fixed cost of safety investment k, and low safety if otherwise. When deviation to low safety reduces firm j's effective marginal cost ($\alpha\theta z - c < 0$), consumers hold the belief that product j has high safety if the firm charges a price higher than a certain threshold $\widehat{p}$, and low safety if otherwise. Similarly, when deviation to low safety increases firm j's effective marginal cost ($\alpha\theta z - c > 0$), consumers hold the belief that product j has high safety if the firm charges a price lower than $\widehat{p}$, and low safety if otherwise.

Finally, if $p_j \geq p^e + t$, consumers would not buy product j no matter what their belief is, and hence firm j has zero output in period 1. It can also be verified that a high-safety firm would never prefer zero output to positive output. For convenience, we thus assume that, if $p_j \geq p^e + t$, consumers in period 1 hold the belief that product j has low safety, and consistently, consumers and other firms in period 2 will hold the same belief that product

j has low safety.

Let α^N be such that $\Delta(N) = k - \frac{\alpha^N \theta z - c}{N}$. One can verify that $\widehat{p} \neq p^e$ if $\alpha > \alpha^N$,while $\widehat{p} = p^e$ if $\alpha = \alpha^N$. Furthermore, if $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, we have $\Delta(N) \geq k - \frac{\alpha\theta z - c}{N} = k$, so that $B(p_j, p^e) = d$ for any $p_j \in [p^e - t, p^e + t)$. The following lemma summarizes the above characterization of consumer beliefs and further shows that high-safety equilibrium does not exist if $\alpha < \alpha^N$.

Lemma 4 *The high-safety equilibrium does not exist if $\alpha < \alpha^N$. If $\alpha \geq \alpha^N$, for $p_j \geq p^e + t$, consumer belief in both periods satisfies $B(p_j, p^e) = D$; for $p_j < p^e - t$, consumer belief in period 1 satisfies $B(p_j, p^e) = B(p^e - t, p^e)$; and for $p_j \in [p^e - t, p^e + t)$, consumer belief takes the following form: (1) Suppose that $\alpha\theta z - c = 0$. Then $B(p_j, p^e) = d$. (2) Suppose that $\alpha\theta z - c < 0$. Then there exists some unique $\widehat{p}$, as defined in (2), such that $B(p_j, p^e) = d$ if $p_j \geq \widehat{p}$ but $B(p_j, p^e) = D$ if $p_j < \widehat{p}$; also $\widehat{p} \leq p^e$, with strict inequality if and only if $\alpha > \alpha^N$. (3) Suppose that $\alpha\theta z - c > 0$. Then there exists some unique $\widehat{p}$, as defined in (2), such that $B(p_j, p^e) = d$ if $p_j \leq \widehat{p}$, but $B(p_j, p^e) = D$ if $p_j > \widehat{p}$; also $\widehat{p} \geq p^e$, with strict inequality if and only if $\alpha > \alpha^N$.*

Based on Lemma 4, when $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta z - c \neq 0$, if firm j charges a price that is in a small neighbourhood of p^e, consumers will hold the belief that product j has high safety. With a high-safety product, if firm j chooses a price much higher or lower than p^e so that consumers would believe that it has deviated to low safety, then its profit in period 1 would be lower. Thus, in any high-safety equilibrium in period 1, the optimal price for firm j solves the following problem:

$$\max_{p_j} [p_j - c - \alpha\theta(d - \delta^*)] \frac{t - p_j + p^e}{Nt}.$$

We then have the optimal price for firm j satisfying

$$p^e = \frac{p^e}{2} + \frac{t + c + \alpha\theta(d - \delta^*)}{2} \equiv \frac{p^e}{2} + \frac{p^*}{2},$$

which immediately implies

$$p^e = p^* = t + c + \alpha\theta(d - \delta^*).$$

Therefore, whenever $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$, or $\alpha > \alpha^N$ and $\alpha\theta z - c \neq 0$, if high-safety equilibrium exists, it must be unique with all firms charging p^* in period 1.

However, when $\alpha = \alpha^N$ and $\alpha\theta z - c \neq 0$, without further refinement, we can have a continuum of high-safety equilibria. For example, under the above consumer belief, it can be verified that, with $\alpha\theta z - c < 0$, there can exist a continuum of equilibria with prices p^e greater than or equal to p^*.[22] Similarly, when $\alpha\theta z - c > 0$, there may also exist a continuum of equilibria with prices lower than or equal to p^*. Notice that this multiplicity of high-safety equilibria only occurs when $\alpha = \alpha^N$. But since the unique equilibrium price is p^* when $\alpha > \alpha^N$, one might consider the equilibrium with $p^e = p^*$ when $\alpha = \alpha^N$ as being more robust, in the sense that the equilibrium strategy when $\alpha = \alpha^N$ is stable with respect to a small perturbation of the environmental variable α. We formalize this idea by considering the following "stable equilibrium" refinement when there are multiple equilibria:

Definition 1 *An equilibrium associated with a specific liability level α in our model is stable if, in period 1, its equilibrium price is the limit of equilibrium prices for liability levels that are larger than but arbitrarily close to α.*

This concept of stable equilibrium shares some similar motivation as the trembling-hand perfect equilibrium (Selten, 1975), even though they are not the same.[23] In what follows, we shall focus on the stable equilibrium when there exist multiple equilibria. Therefore, for any $\alpha \geq \alpha^N$, the unique equilibrium price in period 1 is $p^e = p^*$.

Combining the above analysis for period 1 with that for period 2, we see that when the (stable) high-safety equilibrium exists, it is unique with all firms charging $p^* = q^*$ in both periods and earning total profits $\frac{2t}{N} - k$ in two periods together. We next characterize the conditions for the existence of the equilibrium. Given Lemma 4, any high-safety equilibrium does not exist if $\alpha < \alpha^N$. So suppose that $\alpha \geq \alpha^N$. To sustain the high-safety equilibrium, we need to consider two types of deviations by a firm: first, deviating to low safety but charging a price leading consumers to believe that it has high safety; and second, deviating to low safety and charging a price leading consumers to believe that it has low safety. For the first type of deviation, Lemma 4 implies that, in order for consumers in period 1 to believe that it has high safety, the deviating firm has to choose a price less than $p^* + t$ and therefore has positive output, which reveals its true product safety to consumers and other firms in period 2. Then, as long as $\alpha \geq \alpha^N$, based on our earlier analysis of consumer belief, the firm will have no incentive to deviate to low safety.

Now consider the second type of deviation. At the proposed equilibrium where all firms choose high safety and charge p^*, if a firm, say firm 1, deviates to low safety and charges a price such that $B(p_1, p^*) = D$, the demand for product 1 in period 1 would be

$$\frac{2}{N} \max\{\frac{t - p_1 + p^* - (1-\alpha)\theta z}{2t}, 0\},$$

provided $\frac{t-p_1+p^*-(1-\alpha)\theta z}{2t} < 1$.

Notice that, given $B(p_1, p^*) = D$, the deviating firm's profit in period 2 would be the same, no matter whether it has positive or zero output in period 1.[24] In period 1, ignoring the constraint $B(p_1, p^*) = D$, firm 1's optimal price after deviation therefore maximizes its period 1 profit:

$$\max_{p_1}\{[p_1 - \alpha\theta(D-\delta^*)]\frac{2}{N}\max\{\frac{t-p_1+p^*-(1-\alpha)\theta z}{2t}, 0\}\}.$$

The optimal deviating price in period 1 is $\tilde{p}_1 = t + \alpha\theta(D-\delta^*) - \frac{\theta z - c}{2}$. Given A3 ($t > t_1 = \frac{\theta z - c}{2}$), if $B(\tilde{p}_1, p^*) = D$, firm 1 has positive output and its deviating profit in period 1 is

$$[\tilde{p}_1 - \alpha\theta(D-\delta^*)]\frac{2}{N}\frac{t - \frac{\theta z - c}{2}}{2t} = \frac{(t-t_1)^2}{Nt}.$$

Accordingly, if $B(\tilde{p}_1, p^*) = D$, the difference between firm 1's equilibrium profit and deviation profit in two periods is

$$(\frac{2t}{N} - k) - [\frac{(t-t_1)^2}{Nt} + \frac{(t-t_2)^2}{Nt}] = [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) - k.$$

Hence, if $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) \geq k$, firm 1 would not deviate to low safety with a deviating price such that $B(p_1, p^*) = D$.

It follows that a sufficient condition for the existence of the high-safety equilibrium is: $\alpha \geq \alpha^N$ and $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) \geq k$. In the appendix, we show that this condition is also necessary. Thus, we have

Proposition 1 *There exists a unique equilibrium where all firms produce the high-safety* *product and charge* $p^* = t + c + \alpha\theta(d - \delta^*)$ *in both periods if and only if*

$$\alpha \geq \alpha^N \text{ and } [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) \geq k. \tag{3}$$

It can be verified that condition (3) is consistent with (A2). Define the socially optimal liability as α^*. We prove in the appendix the following corollary of Proposition 1:

Corollary 1 *There exist two cut-off values* k_1, k_2, *with* $0 \leq k_1 < k_2$ *and both independent of* α. *If* $k \leq k_2$, *the high-safety equilibrium exists, with* $\alpha^* = 0$ *when* $k \leq k_1$ *and* $\alpha^* = \alpha^N \in (0, 1]$ *satisfying* $\frac{\alpha^*\theta z - c}{N} + \Delta(N) = k$ *when* $k_1 < k \leq k_2$. *If* $k > k_2$, $\alpha^* = 0$, *and the high-safety equilibrium does not exist.*

Corollary 1 characterizes the socially optimal liability that ensures the existence of the high-safety equilibrium. We next examine how the optimal liability depends on competition, considering in turn two alternative measures of competition intensity: product differentiation between firms and the number of firms in the market.

3.2 PRODUCT LIABILITY AND PRODUCT DIFFERENTIATION

In our spatial model of oligopoly, consumers' unit transportation cost (t), which indicates their preference heterogeneity or the degree of horizontal product differentiation, is a natural measure of the intensity of competition. When t decreases, consumers are less heterogeneous, which reduces product differentiation and lowers equilibrium market prices. The result below shows that the optimal liability generally increases when competition is more severe in the sense that t decreases.

Proposition 2 *Holding all other parameter values constant, there exist two cut-off values*

$t_L < t_H$ such that: (i) when $t < t_L$ or $t \geq t_H$, the socially optimal liability $\alpha^* = 0$, and (ii) when $t_L \leq t < t_H$, α^* is positive and strictly decreases in t.

Thus, product liability and market competition tend to be complements, when competition intensity is measured by the degree of product differentiation. When there is less product differentiation, firms compete more aggressively, resulting in lower profits in both periods and lower benefit from being known as a high-safety producer in period 2. Then, if a firm deviates to low safety, its "reputation loss" in period 2 would be smaller. Consequently, to sustain the high-safety equilibrium, product liability should be increased to raise the deviation cost.

Furthermore, when product differentiation is sufficiently small ($t < t_L$), competition is so fierce that a firm's reputation loss from deviation becomes too small to overcome the deviation gain. Therefore, under any liability level, the high-safety equilibrium does not exist. Then, the socially optimal liability should be zero in order to maximize consumers' precaution incentives. On the other hand, when product differentiation is sufficiently large ($t \geq t_H$), the reputation loss from deviation is high enough to maintain a firm's investment incentive. Then, the high-safety equilibrium exists under any liability level, and thus again the socially optimal liability is zero.

3.3 PRODUCT LIABILITY AND THE NUMBER OF COMPETITORS

We next examine how the optimal liability may vary with the number of competitors. A change in the number of competitors affects not only the reputation loss from deviation, but also each firm's equilibrium output level that has a non-monotonic impact on the deviation gain in period 1. The net effect of a change in the number of competitors on the optimal liability can thus be ambiguous. For convenience, we shall treat N as a continuous variable in our analysis. As shown in the online appendix, the following lemma states a monotonic relationship between the reputation loss from deviation and the number of competitors.

Lemma 5 *At the high-safety equilibrium, a firm's reputation loss from deviating to low safety,* $\Delta(N)$, *strictly decreases in* N.

From Corollary 1, when the socially optimal liability is positive, it satisfies

$$\Delta(N) = k - \frac{\alpha^*\theta z - c}{N},$$

where the term on the right-hand side is the potential gain from deviating to low safety but charging the equilibrium price in period 1, which includes saving of the fixed investment cost and profit variation due to the marginal cost change $(\alpha^*\theta z - c)$. If $\Delta(N) \geq k$, then $\alpha^*\theta z - c \leq 0$, that is, deviation reduces the effective marginal cost; and if $\Delta(N) < k$, then $\alpha^*\theta z - c > 0$, implying higher effective marginal cost from deviation. Therefore, this deviation gain in period 1 may decrease or increase in N, depending on the sign of $\alpha^*\theta z - c$.

Define

$$\Psi(N) = \frac{d[N\Delta(N)]}{dN}.$$

In the appendix, we show that $N\Delta(N)$ is a concave function in N. The result below demonstrates that, holding all other parameters constant, the optimal liability may vary non-monotonically with the number of competitors.

Proposition 3 *Suppose that* $\alpha^* > 0$ *for* $N \in [N_1, N_2]$, *with* $2 \leq N_1 < N_2 \leq \overline{N}$. *(i) If*

$\Psi(N_2) < k < \Psi(N_1)$, *then there exists some* $\widehat{N} \in (N_1, N_2)$ *such that for any* $N \in [N_1, N_2]$, *the optimal liability* α^* *decreases in* N *for* $N < \widehat{N}$ *and increases in* N *for* $N > \widehat{N}$. *(ii) If* $k \leq \Psi(N_2)$, *then* α^* *decreases in* N *for any* $N \in [N_1, N_2]$. *(iii) If* $k \geq \Psi(N_1)$, *then* α^* *increases in* N *for any* $N \in [N_1, N_2]$.

If a firm deviates to low safety and still charges the equilibrium price, it benefits from saving the fixed cost of investment and the variable production cost in period 1, but suffers from the extra liability cost in period 1 and the reputation loss in period 2. A firm would sell a high-safety product only when the reputation loss is larger than the deviation gain in period 1. An increase in the number of competitors always reduces the reputation loss from deviation (due to each firm's lower output), while the deviation gain in period 1 may vary non-monotonically with the number of competitors. Now, to see the intuition behind part (i) in Proposition 3, consider two cases when $\Psi(N_2) < k < \Psi(N_1)$:

First, if the number of competitors (N) is relatively small, reputation loss from deviation is large, so that the optimal extra liability cost to sustain the high-safety equilibrium is smaller than the variable production cost (i.e., $\alpha^* \theta z < c$). In this case, deviation not only saves a firm's fixed cost of investment, but also provides a "reward" of lower effective marginal cost. A firm's total deviation gain in period 1 then decreases in N, as an increase in N reduces each firm's output level. Thus, when N increases, while the decreased reputation loss raises the firms' incentive for deviation, the decreased deviation gain in period 1 reduces it. For small enough N, the latter effect dominates, and hence the optimal liability decreases in N. In this case, competition and product liability are substitutes to achieve high product safety and also efficiency. However, for larger N, the former effect dominates, and hence the optimal liability increases in N. In this case, competition and product liability become complements to achieve high product safety and efficiency.

Second, if the number of competitors is large enough, the reputation loss from deviation is small, so that the optimal extra liability cost sustaining the high-safety equilibrium is larger than the variable production cost. In this case, deviation saves a firm's fixed cost, but generates a "penalty" of higher effective marginal cost. Thus, a deviating firm's total gain in period 1 increases in the number of competitors. When N increases, both the decreased reputation loss and the increased deviation gain in period 1 raise the firms' incentive for deviation. Therefore, the optimal liability must be increased to maintain the firms' investment incentive. In this case, competition and product liability continue to be complements to achieve high product safety and efficiency.

To see the intuition behind (ii) and (iii) of Proposition 3, notice that if k is small enough, then $\alpha^*\theta z$ is much smaller than c. Thus, a higher N significantly lowers a firm's deviation gain in period 1 and this effect always dominates the reputation effect. Hence α^* is optimally set lower. However, if k is large enough, then to sustain the high-safety equilibrium, liability must be high enough such that deviation increases the effective marginal cost. In this case, a higher N raises the firm's deviation incentive and hence α^* needs to be higher.

The numerical example below illustrates the possibility that α^* may first decrease and then increase in N as in Proposition 3.

Example 1 *Let* $\theta z = 2, t = 1, c = 1$ *and* $k = 0.01$.[25] *In addition, let* $\max_\delta\{\theta\delta - \phi(\delta)\} < 1$. *Then, we find that* $\alpha^* > 0$ *for any* $N \in [2, \overline{N}]$, *and the socially optimal liability* α^* *first decreases in* N *when* $N < \widehat{N} = 6$ *and then increases in* N *when* $6 < N < \overline{N}$, *as shown in*

Figure 2 below:

<<COMP: Place Figure 2 about here>>

The following corollary considers one special case of Proposition 3. When the variable cost of providing high safety becomes zero, the socially optimal liability always increases with the number of competitors.

Corollary 2 *If $c = 0$ and $\alpha^* > 0$ for $N \in [N_1, N_2]$, with $2 \leq N_1 < N_2 \leq \overline{N}$, then α^* strictly increases in N for any $N \in [N_1, N_2]$.*

The results in this section can lead to useful policy implications. For example, given the number of competitors, in a mature market with relatively small product differentiation, product liability should be large enough to sustain firms' incentives in making safety investments; in contrast, when the market features larger product differentiation possibly due to new innovation or different technologies, as implied by Proposition 2, product liability can be reduced, or equivalently, courts can adopt a higher standard for evidence in deciding whether a firm is liable for consumer damage. This implication is aligned with the general idea that more protection (here less product liability) should be given to firms in markets with new innovation or different technologies, but in our paper, this result is based on the relationship between product liability and competition intensity as measured by product differentiation.

For another example, in choosing product liability, courts and regulators should consider the number of competitors as well as firms' cost structure in making safety investment, although it may be difficult to set the optimal liability level based on the number of competitors. In practice, firms can adopt various technologies or methods to improve product safety and reduce consumer damage. If firms can take R&D projects to improve product safety and warn consumers about the potential harm, then the relevant costs for firms are mainly fixed costs instead of variable costs. In such cases, as shown in Corollary 2, product liability and competition as measured by the number of competitors are complements. In industries with more competitors or when new entrants arrive, liability should be increased to motivate firms' R&D effort. In contrast, if firms not only incur fixed R&D costs but also add safety devices with variable costs to increase product safety, then product liability and competition can be either substitutes or complements, as shown in Proposition 3. In particular, if the number of competitors is small enough (for example, in a new market), it is likely that product liability and competition are substitutes. In such cases, liability can be reduced when a new firm enters the market.

Notice that in our model product liability does not affect the equilibrium profit and therefore would not change a firm's entry decision. In more general settings where firms' profits depend on liability, however, changing product liability can have an extra effect on entry decisions. The relationship between product liability and endogenous entry decisions can be an interesting topic for future research.

4. DISCUSSION

We have conducted our analysis in a variant of the spokes model that extends the classic Hotelling duopoly. We wish to allow product differentiation between firms in the market, for which the Hotelling model is known to have very desirable features. To extend Hotelling to an oligopoly with any number of firms, one motivation to use the spokes model instead of, say, the circle model (Salop, 1979), is that when only one firm deviates from safety investment, in the subgame of period 2 there is naturally a two-price equilibrium that is easy to characterize in the spokes model, because all $N-1$ firms remain symmetric to each other and with respect to the deviating firm. By contrast, in the circle model, firms are not symmetric as they are located farther away from the deviating firm in each direction, and hence in the equilibrium of period 2 following a firm's deviation there could be more than $\frac{N}{2}$ distinctive prices, which would be extremely difficult to characterize for an arbitrary N.[26]

While our spatial model is relatively simple and intuitive to work with, it imposes specific structures that can be restrictive.[27] We have also made other simplifying assumptions in the model to make the analysis tractable. In this section, we first discuss several alternative formulations to shed light on the robustness of our results. Specifically, we consider in turn an alternative demand system, an alternative formulation on consumer precaution efforts, and an alternative assumption on observability in period 2. Our main insights concerning the relationship between competition and product liability remain valid in these alternative settings. Specifically, competition and product liability can be either complements or substitutes in their roles to promote product safety, and the nature of the relationship may depend on how competition is measured. Furthermore, the relationship may be non-monotonic. An online appendix provides the detailed analysis corresponding to the first three subsections below. At the end of this section, we also discuss some alternative liability rules.

4.1 ALTERNATIVE CONSUMER DEMAND WITH VARIABLE TOTAL OUTPUT

The demand in our main model, as in other spatial models of competition, has the restrictive feature that in equilibrium the market is fully covered with a fixed total output. This removes the extensive margin that would also need to be taken into account in determining the socially optimal product liability. We now briefly explain that our main results concerning competition and product liability can also hold in an alternative model of consumer demand with variable total output (Daughety and Reinganum, 2008b), which is derived from the qausi-linear quadratic utility function of a representative consumer.

Following Daughety and Reinganum (2008b), consider a single consumer who consumes Q_j of product j, $j = 1, ...N$, with utility function

$$U(Q_1, ..Q_N) = \sum_j [A - (1-\alpha)\theta(B_j - \delta_j) - \phi(\delta_j)]Q_j - \frac{1}{2}(\sum_j \beta Q_j^2 + \sum_j \sum_{i \neq j} \gamma Q_j Q_i),$$

where $B_j = \{D, d\}$ is the consumer's belief about damage level caused by product j and $\gamma > 0$ is the degree of product substitution between any two products. Assume that $\beta > \gamma$ and A is large enough such that there is always positive output for each product. This utility function leads to the following demand for any product j

$$\begin{aligned} & Q_j(p_1, ..p_N; B_1, ...B_N) \\ = \; & A' - b[(1-\alpha)\theta(B_j - \delta_j) + \phi(\delta_j)] + g \sum_{i \neq j}[(1-\alpha)\theta(B_i - \delta_j) + \phi(\delta_j)] - bp_j + g \sum_{i \neq j} p_i \ , \end{aligned}$$

where $A' = \frac{A}{\beta + (N-1)\gamma}$, $b = \frac{[\beta + (N-2)\gamma]}{(\beta - \gamma)[\beta + (N-1)\gamma]}$, and $g = \frac{\gamma}{(\beta - \gamma)[\beta + (N-1)\gamma]}$.

The full-fledged analysis based on the general demand function becomes rather complicated. For simplicity, we consider two special scenarios investigating the relationship between liability and the different measures of competition, N and γ, respectively.

First, under the assumption that $\beta = 2$ and $\gamma = 1$, we examine the relationship between the optimal liability and the number of competitors (N). At the high-safety equilibrium, if a firm deviates to low safety, it gains profit in period 1 while suffers from reputation loss in period 2. The deviation gain in period 1 decreases in α, but varies non-monotonically in N, depending on the change in the effective marginal cost ($\alpha\theta z - c$). In contrast, the reputation loss is independent of liability α and decreasing in N. Notice that these effects are the same as those in our main model. Following similar analysis in Section 3, we can show that the optimal liability, whenever positive, changes non-monotonically when N increases, possibly first decreasing and then increasing.

Second, under the assumption that $N = 2, \beta = 1$,and $\gamma \in (0, 1)$, we explore the relationship between the optimal liability α^* and the degree of product substitution (γ). While we are unable to derive analytically the general relationship, in the numerical examples we have analyzed, we find that whenever the optimal liability $\alpha^* > 0$, it increases in γ. That is, when competition is more fierce due to more product substitution, or equivalently, less product differentiation, the optimal liability increases to maintain the firms' investment incentives, same as in our main model.

Therefore, as in our main model, the relationship between competition and the optimal liability is subtle, depending on how competition is measured.

4.2 CONSUMER PRECAUTION CONTINGENT ON PRODUCT SAFETY

In our model, consumer precaution is assumed to reduce the harm of a faulty product by a certain amount (δ), and the precaution effort is independent of the product safety level. An alternative assumption could be that consumer precaution can reduce the damage by a fraction. Then, in equilibrium consumer precaution effort will depend on their belief about product safety. Nevertheless, our main results still hold: when competition increases due to less product differentiation, the optimal liability tends to become higher; whereas when competition increases due to a larger number of competitors, the optimal liability can vary non-monotonically.

Suppose that a consumer can reduce the damage by a fraction of $1 - \eta \in [0, 1)$. This is equivalent to assuming that a consumer can reduce the probability of harm to $\eta\theta$. Note that lower η implies less expected damage. Each consumer's precaution cost is $\varphi(\eta)$, which is strictly decreasing and convex, with $\varphi'(0) = -\infty$.

A consumer's optimal precautions, when she believes that the product has high or low safety, respectively are $\eta(d, \alpha)$ or $\eta(D, \alpha)$ and solve:

$$\max_{\delta} \left\{-(1-\alpha)\theta\eta d - \varphi(\eta)\right\}; \qquad \max_{\delta} \left\{-(1-\alpha)\theta\eta D - \varphi(\eta)\right\}.$$

Clearly, $\eta(D, \alpha) \neq \eta(d, \alpha)$. That is, consumer precaution depends on her belief about product safety. To simplify notations, define the sum of consumer damage and precaution cost as $\Phi(D, a) \equiv \theta\eta(D, \alpha)D + \varphi(\eta(D, \alpha))$ and $\Phi(d, a) \equiv \theta\eta(d, \alpha)d + \varphi(\eta(d, \alpha))$. Similar to the analysis in Section 3, it can be shown that, whenever the optimal liability sustaining the high-safety equilibrium is positive (i.e., $\alpha^* > 0$), it satisfies the following condition:

$$\Omega(N, \alpha^*) = k - \frac{1}{N} \min\{\alpha^* \theta \eta(d, \alpha^*) z - c, t - \frac{(t - \frac{[\Phi(D,\alpha^*) - \Phi(d,\alpha^*)] - c}{2})^2}{t}\}, \tag{4}$$

where the term on the right-hand side is a deviating firm's gain in period 1, which decreases in α^* and t; while $\Omega(N, \alpha^*)$ is the reputation loss, which increases in α^* and t. Then, for the range of t under which $\alpha^* > 0$, when t decreases, to maintain condition (4) , the optimal liability α^* is higher.

Additionally, it can be shown that the reputation loss $\Omega(N, \alpha^*)$ decreases in N, while the deviation gain in period 1 changes non-monotonically in N, depending on the liability level. Thus, as in our main model, the optimal liability α^* can vary non-monotonically in N.

4.3 IMPERFECT OBSERVATION OF PAST DAMAGES

Our main model assumes that consumers in period 2 can observe what happened in period 1. Hence, from damages (and output levels) occurred in period 1, consumers can detect any firm who has deviated to selling a low safety product. While this perfect observability is clearly a strong assumption, our results can continue to hold if our model is extended to allow imperfect observability. In particular, suppose that, with probability $r \in (0, 1]$ all consumers and firms in period 2 observe what happened in period 1, while with probability $1 - r$ no consumer (and no other firm) observes a product's past damage or output level. Such imperfect and symmetric observability could arise due to the costs or imperfect implementation of disclosure regarding market information and consumer damage. Then it can be shown that the socially optimal liability to sustain the high-safety equilibrium (α^*), whenever positive, satisfies

$$r\Delta(N) = k - (2-r)\frac{\alpha^*\theta z - c}{N}. \tag{5}$$

The characterization of the optimal liability in condition (5) is similar to that in Section 3. Therefore, similar to the analysis there, more intense competition due to less product differentiation (i.e., a lower t) tends to increase the optimal liability, while an increase in the number of competitors (N) can change the optimal liability non-monotonically.[28]

4.4 ALTERNATIVE LIABILITY RULES

Our main model focuses on a strict liability rule under which consumers receive compensation equal to a certain fraction of product damages from firms. Consumers and firms thus face a two-sided moral hazard problem that causes inefficiency. Alternatively, when consumers are harmed, if firms will pay liability penalty (possibly to the government) but consumers do not receive any compensation, then consumers will take the socially optimal precaution while liability for firms can induce them to make high-safety investment. While such a scheme to decouple product liability can increase welfare, it may be difficult to implement due to legal constraints, and it may also be unfair to consumers who suffer damages but are not compensated.

More generally, an alternative liability rule to enhance efficiency is negligence with a defense of contributory negligence.[29] If courts can observe firms' safety investment and consumers' precaution levels (in our simple model with two damage levels, a court informed with all environmental parameter values can perfectly infer the players' actions), the following rule can increase welfare: a firm provides full liability compensation to a harmed consumer only if the consumer has taken the socially optimal precaution (i.e. $\delta(0)$ in our model), and furthermore, the firm's liability level can be reduced if the firm has made safety investment. In practice, if courts cannot observe values of the environmental parameters or if more uncertainty about consumer damages exists, it would be difficult to implement such negligence rules. However, it could be interesting for future research to explore the relationship between competition intensity and other liability regimes that can potentially promote product safety and efficiency more effectively.

5. CONCLUSION

This paper has studied the relationship between competition and product liability in their roles to improve product safety and efficiency. We find that this relationship is subtle, depending importantly on what causes the change in competition intensity. Under a given market structure, when competition increases due to less product differentiation, the socially optimal product liability generally increases. In this sense, competition and product liability are complements. On the other hand, as competition increases because the number of competitors rises, the optimal product liability may vary non-monotonically, first decreasing and then increasing.

We have discussed the robustness of our results when several restrictive assumptions of our model are relaxed. Our model also contains other strong assumptions. For instance, our assumption on the safety investment, with only two possible levels and with its only effect as reducing consumer damages when the product malfunctions, is obviously very crude; and our highly-stylized setting in which reputation works is also rather restrictive. While these modelling choices are motivated mainly by analytical tractability, it would be desirable for future research to extend our analysis to other and more general settings.

APPENDIX

The appendix contains proofs for Lemmas 2 and 4; Corollaries 1 and 2; and Propositions 1-3.

Proof of Lemma 2. Consider firm $j's$ pricing decision in period 2. Suppose that all other firms set their prices as q^*. As long as $-t \leq q_j - q^* < t$, the per-period demand for product j becomes $\frac{2}{N(N-1)}(N-1)\frac{t-q_j+q^*}{2t}$. Correspondingly, firm j chooses its price to maximize its period 2 profit

$$\max_{q_j}[q_j - c - \alpha\theta(d-\delta^*)]\frac{2}{N}\frac{t-q_j+q^*}{2t}.$$

The first order condition leads to $q_j = q^* = t + c + \alpha\theta(d-\delta^*)$. It can be verified that the second order condition holds. Correspondingly, each firm's output in period 2 is $\frac{1}{N}$. Therefore, each firm's profit in period 2 is $[q^* - c - \alpha\theta(d-\delta^*)]\frac{1}{N} = \frac{t}{N}$. ■

Proof of Lemma 4 (consumer belief in period 1). Let α^N be such that $\Delta(N) = k - \frac{\alpha^N\theta z - c}{N}$. We first show that high-safety equilibrium does not exist if $\alpha < \alpha^N$. Notice that $\alpha < \alpha^N$ implies $\Delta(N) < k - \frac{\alpha\theta z - c}{N}$. In any high-safety equilibrium, if a firm deviates to low safety but charges p^e, consumers still believe that the firm's product has high safety.

Therefore, the deviating firm has $\frac{1}{N}$ units of output in period 1, but saves the fixed cost k and changes its marginal cost by $\alpha\theta z - c$. Thus, the deviating firm's net gain in period 1 would be $k - \frac{\alpha\theta z - c}{N}$, larger than the reputation loss $\Delta(N)$. Therefore, high-safety equilibrium does not exist.

Now suppose that $\alpha \geq \alpha^N$. In the following, we characterize consumer belief following the logic specified in section 3. Suppose that, in period 1, firm j charges p_j while all other firms charge the equilibrium price p^e. Notice that, whenever $p_j \geq p^e + t$, firm j has zero output no matter what consumer belief is. When $p_j < p^e + t$, given the prior that all products have high safety, firm j's total profit in two periods by choosing high safety is

$$-k + [p_j - c - \alpha\theta(d - \delta^*)]\frac{2}{N}\min\left\{\frac{t - p_j + p^e}{2t}, 1\right\} + \frac{t}{N},$$

where $-k$ is the investment cost, the second term is firm j's profit in period 1, and the last term is its profit in period 2.

Similarly, firm j's total profit in two periods by choosing low safety is

$$[p_j - \alpha\theta(D - \delta^*)]\frac{2}{N}\min\left\{\frac{t - p_j + p^e}{2t}, 1\right\} + \frac{(t - t_2)^2}{Nt}.$$

Therefore, given $p_j < p^e + t$, choosing high safety can bring larger profit to firm j if and only if

$$\Delta(N) \geq k - (\alpha\theta z - c)\frac{2}{N}\min\left\{\frac{t - p_j + p^e}{2t}, 1\right\}. \tag{A1}$$

When $p_j \leq p^e - t$, the right hand side of condition (A1) becomes $k - (\alpha\theta z - c)\frac{2}{N}$, the same as if firm j charges $p^e - t$. Therefore, for any $p_j < p^e - t$, consumer belief satisfies

$B(p_j, p^e) = B(p^e - t, p^e)$. Now for $p_j \in [p^e - t, p^e + t)$, the right hand side of condition (A1) becomes $k - (\alpha\theta z - c)\frac{t - p_j + p^e}{Nt}$. Let $\widehat{p}$ be such that $\Delta(N) = k - (\alpha\theta z - c)\frac{t - \widehat{p} + p^e}{Nt}$. We then consider three scenarios:

(1) Suppose that $\alpha\theta z - c = 0$. Obviously condition (A1) holds if and only if $\Delta(N) \geq k$. Since $\alpha \geq \alpha^N$ and $\alpha\theta z - c = 0$ together imply $\Delta(N) \geq k$, we have $B(p_j, p^e) = d$.

(2) Suppose that $\alpha\theta z - c < 0$. Notice that, in this scenario, the right hand side of condition (A1) decreases in p_j. Therefore, for $p_j \in [p^e - t, p^e + t)$, condition (A1) holds if and only if $p_j \geq \widehat{p}$. That is, consumer belief would be $B(p_j, p^e) = d$ if $p_j \geq \widehat{p}$, and $B(p_j, p^e) = D$ if $p_j < \widehat{p}$. In addition, notice that, when $p_j = p^e$, the right hand side of condition (A1) becomes $k - \frac{\alpha\theta z - c}{N}$. Recall that α^N satisfies $\Delta(N) = k - \frac{\alpha^N\theta z - c}{N}$. Then $B(p^e, p^e) = d$ implies that $\widehat{p} < p^e$ if $\alpha > \alpha^N$,while $\widehat{p} = p^e$ if $\alpha = \alpha^N$.

(3) Suppose that $\alpha\theta z - c > 0$. Notice that, in this scenario, the right hand side of condition (A1) increases in p_j. Therefore, for any $p_j \in [p^e - t, p^e + t)$, condition (A1) holds if and only if $p_j \leq \widehat{p}$. That is, consumer belief would be $B(p_j, p^e) = d$ if $p_j \leq \widehat{p}$, and $B(p_j, p^e) = D$ if $p_j > \widehat{p}$. In addition, notice that, when $p_j = p^e$, the right hand side of condition (A1) becomes $k - \frac{\alpha\theta z - c}{N}$. Recall that α^N satisfies $\Delta(N) = k - \frac{\alpha^N\theta z - c}{N}$. Then $B(p^e, p^e) = d$ implies that $\widehat{p} > p^e$ if $\alpha > \alpha^N$,while $\widehat{p} = p^e$ if $\alpha = \alpha^N$. ■

Proof of Proposition 1. The analysis in the text directly implies that condition (3) is sufficient for the existence of the high-safety equilibrium. Lemma 4 shows that $\alpha \geq \alpha^N$ is a necessary condition. It remains to show that $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) \geq k$ is also necessary. Notice that, if $\alpha\theta z - c = 0$ and $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) < k$, we have $\Delta(N) < k$ and accordingly $\alpha < \alpha^N$, so that the high-safety equilibrium does not exist.

Now suppose that $\alpha \geq \alpha^N$ and $\alpha\theta z - c \neq 0$. We need to prove that whenever $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) < k$, the optimal deviation price $\tilde{p}_1 = t + \alpha\theta(D - \delta^*) - \frac{\theta z - c}{2}$ satisfies $B(\tilde{p}_1, p^*) = D$. That is, if firm 1 deviates to low safety, the optimal deviation price is $\tilde{p}_1$, which leads to consumer belief that firm 1 has low safety. Based on the construction of consumer belief, $B(\tilde{p}_1, p^*) = D$ is equivalent to

$$\Delta(N) + (\alpha\theta z - c)\frac{2}{N}\min\{\frac{t - \tilde{p}_1 + p^*}{2t}, 1\} < k.$$

Notice first that $\tilde{p}_1 - p^* = \alpha\theta z - c - t_1$.Thus, if $\frac{t-\tilde{p}_1+p^*}{2t} \geq 1$, we have $\alpha\theta z - c \leq -t + t_1 \leq 0$ and therefore $\Delta(N) + (\alpha\theta z - c)\frac{2}{N} \leq \Delta(N) + \frac{2}{N}(-t + t_1) < k$, given $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) < k$. That is, if $\frac{t-\tilde{p}_1+p^*}{2t} \geq 1$, $B(\tilde{p}_1, p^*) = D$.

Now suppose that $\frac{t-\tilde{p}_1+p^*}{2t} < 1$. Define $\omega(\alpha) = \Delta(N) + (\alpha\theta z - c)\frac{t-\tilde{p}_1+p^*}{Nt}$. We then have $\omega(\alpha) = \Delta(N) + (\alpha\theta z - c)\frac{(t+t_1)-(\alpha\theta z-c)}{Nt}$, which is strictly concave in $\alpha\theta z - c$. Thus, $\omega(\alpha)$ achieves the maximal value when $\alpha\theta z - c = \frac{t+t_1}{2}$, i.e. $\alpha = \frac{t+t_1}{2\theta z} + \frac{c}{\theta z}$. Now consider two scenarios.

First, suppose that $t \geq 3t_1$. Also recall that $t_1 \equiv \frac{(\theta z - c)}{2}$. Then given $\alpha \leq 1$, we have $\frac{t+t_1}{2} \geq 2t_1 = \theta z - c \geq \alpha\theta z - c$, and therefore

$$\begin{aligned}
\omega(\alpha) &\leq \omega(1) = \Delta(N) + (\theta z - c)\frac{(t + t_1) - (\theta z - c)}{Nt} \\
&= \Delta(N) + \frac{2t_1((t - t_1))}{Nt} \\
&< \Delta(N) + \frac{t^2 - (t - t_1)^2}{Nt} \\
&< k.
\end{aligned}$$

Second, suppose that $t < 3t_1$. Then we have

$$\begin{aligned} \omega(\alpha) &\leq \omega(\frac{t+t_1}{2\theta z} + \frac{c}{\theta z}) = \Delta(N) + \frac{t+t_1}{2} \frac{(t+t_1) - \frac{t+t_1}{2}}{Nt} \\ &= \Delta(N) + (\frac{t+t_1}{2})^2 \frac{1}{Nt} \\ &\leq \Delta(N) + \frac{t^2 - (t-t_1)^2}{Nt} \\ &< k, \end{aligned}$$

where the second inequality follows from $t < 3t_1$ and the assumption A3 ($t > t_1$).

To summarize, in both scenarios, whenever $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) < k$, we have $\omega(\alpha) < k$, or equivalently, $B(\tilde{p}_1, p^*) = D$, which implies that the high-safety equilibrium does not exist. ■

Proof of Corollary 1. Now define $k_1 \equiv \Delta(N) - \frac{c}{N}$ and $k_2 \equiv [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N)$. Obviously, $k_2 > k_1 \geq 0$.

First, suppose that $0 < k \leq k_1$. Recall that α^N satisfies $\frac{\alpha^N \theta z - c}{N} + \Delta(N) = k$, implying $\alpha^N \leq 0$.Therefore, we have $\alpha \geq 0 \geq \alpha^N$ and $[\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) = k_2 > k$. That is, the high-safety equilibrium exists given any liability level. According to Lemma 1, consumer precaution effort is largest when $\alpha = 0$. Therefore, the socially optimal liability is zero.

Second, suppose that $k_1 < k \leq k_2$. It then can be verified that $\alpha^N > 0$. Furthermore, since $k \leq k_2 = [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N) \leq \frac{\theta z - c}{N} + \Delta(N)$, we have $\alpha^N \leq 1$. Given Proposition 1, α^N is the lowest liability sustaining the high-safety equilibrium. According to Lemma 1, the socially optimal liability should be $\alpha^* = \alpha^N$.

Finally, when $k > k_2$, Proposition 1 implies that the high-safety equilibrium does not exist for any liability level. Therefore, based on Lemma 1, the optimal liability is zero. ■

Proof of Proposition 2. Notice that

$$\Delta(N) = \Delta(N,t) = \frac{1}{N}\left[t - \frac{(t-t_2)^2}{t}\right] = \frac{2t_2}{N} - \frac{t_2^2}{Nt},$$

which strictly increases in t. Accordingly, $k_1 \equiv \Delta(N,t) - \frac{c}{N}$ strictly increases in t; and $k_2 \equiv [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N,t) = \frac{2t_1}{N} - \frac{t_1^2}{Nt} + \Delta(N,t)$ also strictly increases in t.

Given any k, let t_H satisfy $k = \Delta(N,t_H) - \frac{c}{N}$ and t_L satisfy $k = [\frac{t_L}{N} - \frac{(t_L-t_1)^2}{Nt_L}] + \Delta(N,t_L)$. It can be verified that $t_L < t_H$.

When $t < t_L$, we have

$$k = [\frac{t_L}{N} - \frac{(t_L-t_1)^2}{Nt_L}] + \Delta(N,t_L) > [\frac{t}{N} - \frac{(t-t_1)^2}{Nt}] + \Delta(N,t) = k_2.$$

Hence, from Corollary 1, the socially optimal liability is zero.

When $t \geq t_H$, we have

$$k = \Delta(N,t_H) - \frac{c}{N} \leq \Delta(N,t) - \frac{c}{N} = k_1.$$

Hence, from Corollary 1, the optimal liability is zero.

When $t_L \leq t < t_H$, we have $k_1 < k \leq k_2$. Hence, from Corollary 1, the socially optimal liability α^* satisfies $\frac{\alpha^*\theta z - c}{N} + \Delta(N,t) = k$. Since $\frac{\alpha^*\theta z - c}{N} + \Delta(N,t)$ strictly increases in t and also increases in α^*, α^* strictly decreases in t. ■

Proof of Proposition 3. Corollary 1 implies that, if and only if $k_1 < k \leq k_2$, the socially optimal liability α^* is positive and equals α^N such that

$$\Delta(N) = k - \frac{\alpha^N \theta z - c}{N}.$$

Suppose that $\alpha^* > 0$ for any $N \in [N_1, N_2]$. Then it is equivalent to show that α^N changes non-monotonically when N increases.

Recall that, $\Delta(N) = \frac{1}{N}[t - \frac{(t-t_2)^2}{t}]$, where $t_2 \equiv t_2(N) = \frac{N-1}{2N-1}(\theta z - c)$. Thus,

$$N\Delta(N) = 2\frac{N-1}{2N-1}(\theta z - c) - \frac{1}{t}[\frac{N-1}{2N-1}(\theta z - c)]^2.$$

We have

$$\frac{d[N\Delta(N)]}{dN} = [2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}]\frac{\theta z - c}{(2N-1)^2} > 0.$$

Furthermore, notice that $\frac{N-1}{2N-1}$ increases in N and therefore $[2 - 2\frac{\theta z - c}{t}\frac{N-1}{2N-1}]$ decreases in N. In addition, $\frac{\theta z - c}{(2N-1)^2}$ decreases in N. Hence, the differentiation $\frac{d[N\Delta(N)]}{dN}$ is positive but decreases in N. That is, $N\Delta(N)$ is strictly increasing and concave in N for $N \geq 2$. Define $\Psi(N) \equiv \frac{d[N\Delta(N)]}{dN}$.

(1) If $\Psi(N_2) < k < \Psi(N_1)$, then define $\widehat{N} = \min\{N : N \in [N_1, N_2] \mid \Psi(N) \leq k\}$. Because $\Psi(N)$ decreases in N, $\widehat{N}$ is well-defined and unique. Recall that, for any N and M, α^N and α^M satisfy $(\alpha^N\theta z - c) + N\Delta(N) = Nk$ and $(\alpha^M\theta z - c) + M\Delta(M) = Mk$.

Notice that, for any $N \in [N_1, \widehat{N})$, $\Psi(N) > k$; and for any $N \in (\widehat{N}, N_2]$, $\Psi(N) < k$. Correspondingly, for any given N and $M \in [N_1, \widehat{N})$ such that $N > M$, we have $N\Delta(N) -$

$M\Delta(M) > (N-M)k$. Therefore,

$$\begin{aligned} & (\alpha^N \theta z - c) - (\alpha^M \theta z - c) \\ = \;& (N-M)k - [N\Delta(N) - M\Delta(M)] < 0. \end{aligned}$$

That is, $\alpha^N < \alpha^M$. α^N decreases in N for $N \in (N_1, \widehat{N})$.

Similarly, for any given N and $M \in [\widehat{N}, N_2]$ such that $N > M$, we have $N\Delta(N) - M\Delta(M) < (N-M)k$. Thus,

$$\begin{aligned} & (\alpha^N \theta z - c) - (\alpha^M \theta z - c) \\ = \;& (N-M)k - [N\Delta(N) - M\Delta(M)] > 0. \end{aligned}$$

That is, α^N increases in N for $N \in [\widehat{N}, N_2]$.

(2) If $k \leq \Psi(N_2) < \Psi(N_1)$, then $\Psi(N) > k$ for any $N \in [N_1, N_2)$. Similar to the above analysis, for any $N \in [N_1, N_2]$, α^N decreases in N.

(3) If $\Psi(N_2) < \Psi(N_1) \leq k$, then $\Psi(N) < k$ for any $N \in (N_1, N_2]$. Similar to the above analysis, for any $N \in [N_1, N_2]$, α^N increases in N. ■

Proof of Corollary 2. Suppose that $c = 0$. According to Corollary 1, whenever $\alpha^* > 0$, it satisfies

$$\Delta(N) = k - \frac{\alpha^* \theta z}{N}. \tag{A2}$$

$k - \frac{\alpha^* \theta z}{N}$ decreases in α^* and increases in N. As shown in Lemma 5, $\Delta(N)$ strictly decreases in N. Therefore, when N increases, to maintain condition (A2), α^* must be larger. ■

Notes

[1]We would like to thank three referees, a co-editor, Albert Choi, Mitchell Polinsky, Kathryn Spier, participants of seminars at the Hong Kong University of Science and Technology, the University of International Business and Economics, and of the 2016 Annual Meeting of the American Law and Economics Association for their valuable comments. Xinyu Hua thanks the Hong Kong Research Grant Council for research support under Grant# 642211.

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[4]A notable exception is Polinsky and Shavell (2010), who argue that market mechanisms and product liability are substitutes as they both can increase product safety.

[5]For example, Mazzeo (2003) shows that airline companies had better on-time performance when there was more competition. Matsa (2011) finds a positive relationship between product quality and competition in the supermarket industry.

[6]Firms and consumers can also specify potential liabilities in their contracts. Wickelgren (2005) analyzes how contractual liability affects a firm's investment incentives.

[7]Baker and Choi (2016) also examine a similar effect with one firm in a repeated game.

[8]Online appendix: http://ihome.ust.hk/~xyhua/CompetitionLiabilityOnlineAppendix.pdf

[9]For other recent applications of the spokes model, see, for examples, Caminal (2010), Rhodes (2011), Caminal and Granero (2012), Germano and Meier (2013), and Reggiani (2014).

[10]Thus, a high-safety product reduces the harm, but not the probability, of product malfunction. For example, a firm can develop and install a high-safety device on the product (such as an airbag in a car), which can reduce consumer damage if the product fails (e.g., if the car is in an accident possibly due to break malfunction). We could alternatively assume that product safety affects the likelihood for consumers to be harmed. Our formulation

with product safety affecting the damage level is more convenient for analysis.

[11]As shown later in this section, given this "additive" nature on potential damage, consumers' optimal precaution effort does not depend on the safety level of the product. This simplifies our analysis. Our main insight from the analysis could still hold if consumer precaution effort were to depend on the safety level.

[12]The main results in this paper can be generalized by allowing punitive damage compensation (i.e. α larger than 1), which, however, would further distort consumers' precaution incentives. Additionally, in practice it can be hard to implement punitive damage compensation.

[13]We shall focus on the symmetric equilibrium where all firms choose high safety and therefore have positive outputs. As will be shown in section 3, if a firm deviates to low safety, it would choose positive output in period 1 as long as product differentiation (t) is not too small.

[14]This implies that consumers in period 2 will also know whether or not a firm produced positive output in period 1.

[15]Our model assumes that courts cannot observe the values of various environmental parameters and therefore cannot infer consumers' precaution levels. In section 4, we shall discuss other regimes such as decoupling liability and negligence rules where product liability depends on consumer precaution.

[16]As shown later in this section, under this assumption, a firm has positive output both along the equilibrium and off the equilibrium path. The online appendix also shows that our main results regarding the relationship between the optimal liability and competition intensity remain valid even if A3 does not hold.

[17]The proof of Lemma 3 is lengthy but similar to the standard analysis of oligopoly pricing games. Therefore, we put the proof in the online appendix.

[18]This result follows from the assumption that consumers are uniformly distributed on the network. Under other consumer distributions, however, liability may affect profits in period 2 off the equilibrium path.

[19]As shown in an earlier version of the paper, under such an off-equilibrium belief, the

high-safety equilibrium that we characterize in this section would in fact uniquely exist, and the main conclusions of the paper would be the same as what we establish under the alternative beliefs described next.

[20]As stated in Dana (2001), "this is the least costly error the firm could have made conditional on the observed price." Such consumer belief has the flavor of forward induction reasoning (see, for example, Kohlberg and Mertens, 1996).

[21]We thank one referee for suggesting this set of off-equilibrium consumer beliefs.

[22]To be more precise, when $\alpha\theta z - c < 0$, under certain parameter values, we have a set of equilibrium prices $[p^*, \overline{p}]$, where $\overline{p}$ is the highest price leading to full market coverage. Assumption A1 in Section 2 implies that the above set is non-empty.

[23]The trembling-hand perfect equilibrium considers perturbed games where the off-equilibrium strategies are played with small probabilities. A strategy set S of all players in the original game is trembling-hand perfect if there is a sequence of perturbed games that converge to the original game in which there is a series of equilibria converging to S.

[24]If firm 1 has positive output, its true product safety will be revealed in period 2; if firm 1 has zero output, given $B(p_1, p^*) = D$, consumers and other firms in period 2 will hold the same belief that product 1 has low safety. In both cases, firm 1's profit in period 2 would be the same as specified in Lemma 3.

[25]For illustration, we choose small k so that Nk is not too large when N becomes large. As discussed in Section 2, if Nk is too large, it is socially efficient not to make investments in product safety. Notice that for convenience we have normalized the size of consumer population to 1. If consumer population size is large, then k can also be large for our numerical example.

[26]The circle model features localized competition with each firm effectively competing against its two neighbours, whereas the spokes model features non-localized competition with each firm competing against all other firms (i.e., competing against the market). See Chen and Riordan (2007) for more discussions comparing these two models.

[27]In the circle model, a decrease in product differentiation has equivalent equilibrium effects as an increase in the number of firms under some conditions (e.g., Stole, 1988; Yang

and Ye, 2008). One may then wonder whether our result that the measure of competition matters to the relationship concerned is specific to the spokes model. Subsection 4.1 below suggests that the result is not specific to the spokes model.

[28] Another potential extension is to consider imperfect and asymmetric observability, where in period 2 all the firms and a fraction of consumers observe what happened in period 1, while the rest consumers do not observe past damage or output levels. In this extension, the analysis under general assumptions of consumer belief would become rather complex and beyond the scope of this paper. In an earlier version of this paper, we show that the main results in this paper still hold given such imperfect and asymmetric observability, under a simple off-equilibrium belief assumption that, when seeing an off-equilibrium price from a firm in period 1, consumers continue to believe that the firm has produced a high-safety product.

[29] There is a large literature on various negligence rules. For examples, see Brown (1973), Polinsky (1980), Shavell (1980), Polinsky and Rogerson (1983), Rubinfeld (1987), and Bar-Gill and Ben-Shahar (2003).

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