Ex ante Investment, Ex post Remedies, and Product Liability

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May 23, 2011

Abstract. A firm can increase product safety through ex ante investment, and can remedy quality problems after sales. An increase in product liability raises returns to ex ante investment through higher consumer demand, but may also negatively affect the investment incentive due to more ex post remedial activities. The trade-off between these "output" and "substitution" effects can result in an inverted U-shaped relationship between product liability and ex ante investment. We find that the firm prefers full liability but consumer welfare can be higher under partial liability. We further identify conditions under which full liability or partial liability is socially optimal.

Running Head: Investment, Remedies, and Liability
JEL Classifications: L15, K13
1. INTRODUCTION

If the use of a firm’s product results in consumer harm due to poor product quality, what should be the firm’s liability? Under the rule of full liability, the firm is required to fully compensate the consumer for the harm; whereas under the rule of partial liability, the expected compensation to the consumer is lower than the consumer’s loss. There has been substantial interest in the product liability issue in economics, primarily because liability rules can have important impacts on firm incentives and economic efficiency. One literature has focused on the effects of product liability on a firm’s incentives for \textit{ex ante} actions. Liability rules can affect a firm’s precaution to ensure product safety (Simon, 1981) or its quality choice (Polinsky and Rogerson, 1983). In addition to product quality choice, product liability also affects a firm’s incentive to disclose quality information through price and other devices (Daughety and Reingen, 1995; 2008a; 2008b).\footnote{Furthermore, in relation to product liability, a firm’s choices between settlement and litigation and between confidential and open settlement may also affect its \textit{ex ante} quality choice (Daughety and Reingen, 2005; 2006).}

With a different focus, another literature has studied the effects of product liability on a firm’s incentives for \textit{ex post} actions after sales. Welling (1991) shows that a firm makes product recalls in order to build its reputation in the market, whereas Marino (1997) argues that mandatory recalls motivate firms to increase product safety. Spier (2011) analyzes a firm’s incentives to buyback unsafe products and finds that the firm offers a lower buyback price than socially desired. Hua (2011) compares strict liability to negligence rules when a firm’s recall not only depends on its own costs/liability but also on consumers’ return incentives.\footnote{For empirical work related to product recalls, see, for examples, Jarrell and Peltzman (1985), Hartman (1987), Hoffer, Rruitt, and Reilly (1988), and Rupp and Taylor (2002).}

In practice, changes in product liability can affect both firms’ \textit{ex ante} investment and \textit{ex post} incentives for remedial actions such as product recalls. For example, in 2008, the US Senate discussed a bill which would give the Consumer Product Safety Commission more power to collect and disclose allegations of injuries. The supporters claimed that the
bill would encourage firms to design safer products. However, others argued that the bill would increase firms’ liabilities too much, which might lead to more product recalls but not safer products. These debates are related to the more general questions: what is the relationship between firms’ ex ante investment in product safety and ex post actions such as product recalls? Would larger product liability motivate firms to increase or decrease ex ante investment before sales? 

In this paper, we bridge and extend the two literatures by analyzing the potential effects of product liability on both ex ante quality investment and ex post remedial actions by the firm. We consider a setting where, before selling its product to heterogeneous consumers, a monopoly firm can make quality investment, and after sales, it learns about the realization of quality and can take actions to remedy the problem if the product is of low quality or is unsafe (so that there might be consumer harm). In addition, in the event that ex post actions by the firm cannot fully correct the problem and a consumer is nevertheless harmed, the consumer will be compensated through a remedy/liability determined by the court under the product liability law. We investigate the interactions between the firm’s ex post actions and ex ante investment, how both activities depend on product liability, and the privately versus socially optimal liability rules. Our analysis identifies two potential effects of product liability on a firm’s ex ante investment incentives: (i) Substitution effect: An increase in product liability increases the firm’s incentive for ex post remedial actions, which, however, may negatively affect the firm’s incentive for ex ante quality investment. (ii) Output effect: Higher product liability increases consumer demand for the product and leads to higher 

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5 Business Insurance, March 17, 2008

6 In the auto industry, for example, manufactures could spend more time in safety design, which typically delays the marketing of new models. Alternatively, they could reduce the ex ante investment, but modify the design after sales. According to some industry observers, firms rushed to promote their new SUVs, even though there had been many warnings about safety issues; these firms later modified the safety design after Ford and Firestone recalled Explorers with tire problems. (Los Angeles Times, March 14, 2010)

7 Thus, there is a distinction between a remedy determined by the court under the liability law to compensate a consumer who is harmed, and the remedy (remedial actions) that a firm takes to correct the quality problem of the product. We sometimes refer to both of them as ex post remedies when no confusion would arise.
equilibrium output given ex ante investment (despite a higher expected marginal cost to the firm and a higher price), which increases the scale economy of investment and hence the incentive for ex ante quality investment. These two opposing effects can lead to an inverted U-shaped relationship between ex ante quality investment and product liability, with the highest investment sometimes obtained when there is less than full liability.

We further show that the firm always prefers full liability, whereas consumer surplus and social welfare may be higher under partial liability. The firm’s preference for full liability arises in our model primarily because of the endogeneity of consumer demand and the commitment role of product liability. In the absence of reputation considerations and of a legal requirement from the liability law, the firm lacks the ex post incentive to take corrective actions (such as product recalls) when its product may cause consumer harm due to low quality, which lowers consumer demand. Full product liability enables the firm to commit to taking such ex post actions and to internalize the loss to consumers, leading to levels of ex post and ex ante activities that are optimal for the firm. If the only way to influence product quality is through ex post remedial actions, consumers would also prefer full product liability. When ex ante investment is also possible, however, consumers may prefer less than full product liability, and the reason is more subtle. A lower liability may increase the firm’s ex ante investment, due to the substitution effect, which improves product quality and possibly leads to higher equilibrium output. Because the monopoly firm is unable to appropriate all the consumer gains from higher product quality and higher output, its ex ante investment tends to be inefficiently low under full liability. Thus partial liability can result in higher consumer surplus and social welfare than full liability.

In particular, holding all else constant, when the potential consumer loss from low quality is large enough, the output effect dominates the substitution effect on ex ante investment, so that full liability motivates higher ex ante investment and more sales than partial liability.

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8Given the equilibrium output, under full liability, the firm’s ex post incentive coincides with that of the society.

9Other studies have also shown that full liability may not be socially optimal, but it is usually because of potential inefficient behavior (or negligence) from consumers (e.g., Brown, 1973; Shavell, 1980). Our result is obtained without consumer moral hazard. We discuss the issue of consumer negligence in Section 5.
In this case, it is socially optimal to implement full product liability. In contrast, when the potential loss is at an intermediate level, the substitution effect can dominate the output effect, in which case partial liability leads to higher ex ante investment and possibly higher consumer surplus and social welfare. Our findings suggest that product liability should consider the relationship between ex ante investment and ex post remedial actions. Ex post efficiency on remedial actions may be sacrificed in order to enhance ex ante efficiency on investment and to mitigate monopoly deadweight loss. Our results also have related policy implications on limited enforcement of warranty, consumer negligence, and punitive damage compensation.

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 examines how liability rules affect the firm’s ex post remedial actions and ex ante quality investment. Section 4 characterizes the profit-maximizing versus socially optimal liability rules. Section 5 discusses the implications of our results for several related policies and also their possible extensions. Section 6 concludes. All proofs are in the appendix.

2. THE MODEL

There are two periods: the ex ante period when a firm sells its product to heterogeneous consumers, and the ex post period when the firm learns additional information about product quality (or safety) and may take remedial actions such as product recalls or product upgrades.

In the ex ante period, before sales, the true product quality is uncertain, with $\theta$ representing the probability that the product is of high quality and $1 - \theta$ the probability that the product is of low quality. The firm can make an investment to increase the high-quality probability, $\theta \in [0, \bar{\theta}]$. The investment cost, $k(\theta)$, is increasing and strictly convex in $\theta$, with $k'(0) = 0$.\(^{10}\) We assume that there is always a non-trivial probability that the product is of low quality. That is, $\bar{\theta} < 1$. This assumption reflects the reality that the firm cannot per-

\(^{10}\) Due to the one-to-one relationship between quality $\theta$ and investment cost $k(\theta)$, for convenience we shall also refer to the firm’s choice of $\theta$ as its quality investment.
fectly control the product quality. Assume that $\bar{\theta}$ is close to one. We further assume that, after the firm’s quality investment, $\theta$ becomes public information. Our main motivation for this admittedly strong assumption is analytical convenience, which allows us to focus on the purpose of this paper. \footnote{Considerations of asymmetric information have been much studied in the literature. The insights from our analysis can still be valid if the firm has private information about $\theta$. For example, suppose that the firm has private information about $\theta$ but can choose to disclose the information. As long as disclosure costs are small, the unraveling result in the literature holds: almost all types of the firm would try to disclose the information, in order to avoid being perceived as the lowest quality type.}

The total mass of consumers is normalized to 1. Consumers’ values for the product, when it is of high quality, are distributed according to c.d.f. $F(v)$, with a corresponding density function $f(v) > 0$ on support $v \in [0, \bar{v}]$. We impose the regularity condition that $f(v)$ is log-concave (i.e., $d^2 \ln f(v) / dv^2 \leq 0$).\footnote{This assumption is satisfied by familiar distributions including uniform, exponential, normal, truncated t-distribution, and extreme-value distribution.} This condition implies that the hazard rate, $\lambda(v) \equiv \frac{f(v)}{1-F(v)}$, is non-decreasing (i.e., $\lambda'(v) \geq 0$). Define the inverse hazard rate as $H(v) \equiv \frac{1}{\lambda(v)}$. If the product is of low quality, it may reduce consumers’ values or cause harm to consumers (independently) with probability $\gamma$, which is also uncertain ex ante and follows a distribution $G(\gamma)$, with a corresponding density function $g(\gamma) > 0$ on $[0, 1]$.\footnote{We can think of a low-quality product as containing some defect, which could harm consumers. A more serious defect has a higher probability ($\gamma$) to cause consumer harm.}

When a consumer is harmed, her value is reduced by $D$.

After making the quality investment, the firm sets its price, and each consumer decides whether or not to purchase the product based on her realized $v$, her expectations about $\theta$, $\gamma$, and ex post remedial actions by the firm if the product is of low quality. The firm’s total sales are denoted by $Q$.

In the ex post period, after sales, the firm learns the true quality of the product. If the product is of low quality, the firm also learns the realization of $\gamma$ (how serious the defect is). We allow the firm’s ex post information to be either private or public. The firm may then take ex post actions to fix the quality problem. We allow the possibility that the ex post actions are not fully effective and can thus fix only a proportion $\beta \leq 1$ of the sold
product, which cost \( C \beta Q \), where \( C \) is the cost of ex post actions required to avoid loss \( D \) from each unit of low-quality sold product. To avoid trivial situations, we further assume that \( D \geq C \geq \varepsilon \) for some fixed \( \varepsilon > 0 \), and that the parameters of the model are such that the firm will optimally choose to produce a positive amount of output (i.e., \( Q > 0 \)).\footnote{If \( C \) is arbitrarily small, then the product quality issue becomes trivial when \( \beta = 1 \), which we allow, since the firm can then fix any quality problem ex post with almost zero cost. If \( D < C \), then the choice of ex post remedial actions becomes trivial, since the firm can never strictly benefit from such actions. The firm will optimally choose \( Q > 0 \), for instance, if for any given \( D, C \) is small enough (which requires \( \varepsilon \) to be small enough) and \( \beta \) is sufficiently close to 1, or if both \( D \) and \( C \) are not too large.}

For the purpose of this paper, we abstract away from the possibility for consumers to take precaution and focus on situations where the firm’s liability is governed by the strict liability rule. If a consumer is harmed, the firm will give compensation \( L \) to the consumer according to product liability. The firm bears "partial liability" if \( L < D \), "full liability" if \( L = D \), and punitive damage compensation if \( L > D \).\footnote{Our analysis can be extended to situations where consumers may take precautions to avoid harm and the firm’s liability is governed by the negligence rules, where the firm may not be liable if courts determine that consumers are negligent. In this case, \( L \) can be interpreted as the firm’s expected compensation to a harmed consumer. See Section 5 for more discussion, where we allow \( \beta \) to be affected by government policies.}

## 3. EX ANTE INVESTMENT AND EX POST REMEDIAL ACTIONS

In this section, we first derive the expected costs of low quality to the firm and to the consumers, which determine the expected social cost. We then derive consumer demand and the firm’s optimal output. One key observation is that the firm’s optimal output can be expressed as a function of \( \theta \) and the ex post social cost per unit of output \((\Delta)\) when quality is low, which allows us to conveniently characterize two benchmarks where either ex ante investment is not considered (so that \( \theta \) is given) or ex post corrections are not available.

We then analyze our general case where the firm can take both ex ante and ex post actions that affect product quality, and examine how the firm’s optimal quality investment depends on the firm’s liability \( L \) and other environmental variables.\footnote{Strictly speaking, we solve for the unique subgame perfect equilibrium of the game. When it will not cause confusion, we simply describe the equilibrium actions of the firm and the consumers as optimal actions.}
In the ex post period, suppose that the firm finds product quality to be low. For ex post social efficiency, a social planner would compare the costs of remedial actions, $C\beta Q$, with the expected ex post social costs if consumers are harmed, $\gamma \beta DQ$. However, the firm will take remedial actions if and only if $C\beta Q < \gamma \beta LQ$, or equivalently, $\gamma > \frac{C}{L}$. Notice that since $\gamma \in [0, 1]$, if $L < C$, the firm will never take ex post actions. In what follows, to consider “interior” solutions where the firm will exert ex post remedial effort with a positive probability, we shall focus on situations where $L \geq C$.

From the ex ante point of view, given low product quality, the firm’s expected ex post cost per unit of output is

$$x = \int_0^{\frac{C}{L}} \gamma LdG(\gamma) + \int_{\frac{C}{L}}^1 [\beta C + (1 - \beta)\gamma L]dG(\gamma),$$

whereas the expected ex post loss for any consumer using a low-quality product is

$$y = \int_0^{\frac{C}{L}} \gamma (D - L)dG(\gamma) + \int_{\frac{C}{L}}^1 (1 - \beta)\gamma (D - L)dG(\gamma).$$

Thus, given low product quality, the expected ex post social cost per unit of output is given by

$$\Delta = \Delta(L) \equiv x + y = \int_0^{\frac{C}{L}} \gamma DdG(\gamma) + \int_{\frac{C}{L}}^1 [\beta C + (1 - \beta)\gamma D]dG(\gamma).$$

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17 With probability $1 - \beta$, the firm’s ex post remedial activities are not effective. Under strict liability, the firm will then have the same expected ex post cost from such a consumer, $\gamma (1 - \beta) LQ$, independent of its ex post actions.

18 $L < C$ is equivalent to $L = C$ in the sense that under both cases the firm will choose ex post actions with zero probability. Conceivably, the equilibrium outcomes under $L < C$ and $L = C$ could still differ, since $L$ may potentially affect the ex ante investment. It turns out, as it will become clear shortly, that in our model all the variables of interest are independent of $L$ if no ex post action will be taken. It is thus without loss of generality for our analysis to focus on situations where $L \geq C$. 

8
Notice that for any $L$,

$$
\frac{d\Delta}{dL} = D \frac{C}{L} g \left( \frac{C}{L} \right) \left( \frac{-C}{L^2} \right) + \left[ \beta C + (1 - \beta) \frac{D}{L} \right] g \left( \frac{C}{L} \right) \frac{C}{L^2}
$$

(4)

$$
= g \left( \frac{C}{L} \right) \frac{C}{L^2} \beta C \left[ 1 - \frac{D}{L} \right].
$$

Simple calculations lead to the following:

**Lemma 1**

(i) $d \Delta(L)/dL \leq 0$ if $L \leq D$, and, ex post, it is socially efficient to have $L = D$.

(ii) $\Delta(L)$ increases in $D$ and in $C$ but decreases in $\beta$. 

If the firm bears full liability $L = D$, it will make the socially efficient decision on ex post actions. If $L < D$, then there will be too little effort relative to the socially desired; if $L > D$, there will be too much. Note that $L > D$ includes punitive damage compensation. Although in theory punitive compensation can be implemented, in practice punitive compensation often cannot be too large perhaps because firms can seek for bankruptcy protection. For convenience, we restrict our analysis to the case with $L \leq D$.\(^\text{19}\)

Given the anticipated ex post cost, a consumer will buy the product ex ante if and only if her value is large enough:

$$
v - p - (1 - \theta)y \geq 0.
$$

Correspondingly, the total demand for the firm’s product is

$$
Q = 1 - F(p + (1 - \theta)y),
$$

or, the inverse demand is

$$
p = F^{-1}(1 - Q) - (1 - \theta)y.
$$

\(^{19}\)In Section 5, we will discuss the possibility of allowing $L > D$. For any given $L' > D$, as long as $L'$ is not too large, there exists a certain level $L < D$ which leads to the same ex post social cost, that is, $\Delta(L) = \Delta(L')$. As it will become clear shortly, the firm’s ex ante quality investment only depends on $L$ through the expected unit social cost $\Delta$, and consequently it is without loss of generality to assume $L \leq D$ as long as $L$ cannot be much higher than $D$. 

9
Given the ex ante quality investment $\theta$, the firm chooses $Q$ to maximize its profit

$$
\pi(\theta) = \max_{Q \leq 1} Q [p - (1 - \theta)x] = \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)(x + y)]
$$

$$
= \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)\Delta(L)].
$$

(6)

Under the monotone hazard rate, it is easy to verify that the above objective function is concave. The first order condition is

$$
F^{-1}(1 - Q) - \frac{Q}{f[F^{-1}(1 - Q)]} = (1 - \theta)\Delta(L).
$$

(7)

Define

$$
t = F^{-1}(1 - Q),
$$

(8)

then $t$ monotonically decreases in $Q$, and equation (7) becomes

$$
t - \frac{1 - F(t)}{f(t)} = (1 - \theta)\Delta(L).
$$

(9)

By definition, $t$ is the value for the marginal consumer who is indifferent between purchasing and not purchasing the product. Notice that our assumption $Q > 0$ is satisfied as long as $(1 - \theta)\Delta(L)$ is not too large. Since $k'(0) = 0$ by assumption, we also have $\theta > 0$ in equilibrium.

Define the firm’s profit-maximizing output as $Q(\theta, L)$. Note that given $\theta$, liability $L$ affects $Q$ only through the change of ex post unit social cost $\Delta(L)$. Given $\theta$, $t$ in equation (9) increases in $\Delta(L)$, which, from (8) and from $\Delta'(L) \leq 0$, in turn implies that $Q(\theta, L)$ increases in $L$. Notice that, given $\theta$, the expected total social cost under output $Q$ is $(1 - \theta)Q\Delta(L)$, the same as the reduction to the firm’s maximum profit. Thus, given $\theta$, the firm fully internalizes the social cost from low quality when setting its price. A higher liability $L$ increases each consumer’s expected utility from the product, which increases consumer demand; but it also increases the firm’s expected marginal (ex post) cost from selling the product. These two effects happen to exactly offset each other. However, $\Delta(L)$ decreases in $L$ (for $L < D$), and thus the firm’s optimal output increases in $L$ through $\Delta(L)$. 

10
Furthermore, by the envelope theorem,
\[
\frac{d\pi (\theta)}{dL} = -Q (1 - \theta) \frac{d\Delta}{dL} = 0
\]
when \(L = D\). That is, if there were no ex ante investment on quality (so that \(\theta\) is given), the firm’s profit would be maximized if there is full liability \((L = D)\). We thus have:

**Lemma 2** Given ex ante quality investment \(\theta\), the expected cost from low quality is the same for the firm as for the society, and the profit-maximizing liability is \(L = D\), same as the ex post socially efficient liability.

The firm’s ex ante quality investment is determined by the following problem:

\[
\max_{\theta} Q(\theta, L)[F^{-1}(1 - Q(\theta, L)) - (1 - \theta)\Delta(L)] - k(\theta).
\]

From the envelop theorem, the optimal \(\theta\) satisfies

\[(10) \quad Q(\theta, L)\Delta(L) - k'(\theta) = 0.\]

Define the firm’s optimal ex ante investment as \(\hat{\theta}(L)\).\(^{20}\) Note that liability \(L\) affects \(Q\) and \(\theta\) only through the change of ex post social cost \(\Delta\). Also define the firm’s highest possible ex ante investment as \(\hat{\theta}\) for all possible \(\Delta\), which is independent of \(D, C, \beta,\) and \(L\).

If the firm has no opportunity to take ex post actions, the unit ex post social cost \(\Delta\) from low quality would be independent of \(L\). The firm’s liability would then be merely a transfer between the firm and consumers. If the firm bears larger liability, the firm’s ex post cost would be increased by a certain amount; at the same time, consumers’ willingness to pay would be increased by the same amount. The effects of these two changes on the firm’s profit exactly cancel each other, as \(\pi (\theta)\) is unchanged when \(\Delta\) is unchanged. This intuition, formalized in the proof in the appendix, leads to the following:

\(^{20}\) Although we have written the firm’s choice of \(\theta\) as \(\theta (L)\), \(\theta\) depends on \(L\) indirectly through the change of \(\Delta\).
Lemma 3 If the firm could not take ex post remedial actions, then its optimal choice of θ, the equilibrium output, and the equilibrium social welfare would all be independent of liability L.

Note that, if \( L \leq C \), the firm would not take any ex post remedial action. Thus, the above lemma implies that, for any \( L \leq C \), the firm’s output and ex ante investment are independent of liability. Now consider our general case where θ is determined endogenously when both ex ante investment and ex post actions are possible. From condition (10), we see that a reduction in \( \Delta(L) \) can either increase or decrease θ, depending on how \( Q(\theta, L)\Delta(L) \) varies with θ. Therefore, with \( \Delta'(L) \leq 0 \), the firm’s optimal ex ante quality investment, \( \theta(L) \), may increase, decrease, or be non-monotonic in L, as in the following:

Proposition 1 For any given \( D \), holding all else constant, there exists a unique value \( \hat{L} \in [C, D] \) such that both \( \theta(L) \) and \( Q(\theta, L) \) increase in \( L \) if \( C \leq L < \hat{L} \); and \( \theta(L) \) decreases in \( L \) if \( L > \hat{L} \). Furthermore, there exists a unique value \( \tilde{D} > C \) such that \( \tilde{L} = C \) if \( D \leq \tilde{D} \) and \( \tilde{L} > C \) otherwise.

A change in product liability can potentially have two opposing effects on ex ante quality investment. Given output \( Q \), there is a "substitution effect": more ex post remedial actions due to larger \( L \) reduce the unit ex post social cost \( \Delta \), which in turn leads to lower ex ante investment (so \( \theta \) is lower). On the other hand, there is an "output effect": lower ex post social cost \( \Delta \) leads to a larger quantity of sales \( Q \). From condition (10), a larger \( Q \) increases the return to investing in \( \theta \), because an improvement on product quality now applies to a larger output scale. Therefore, the output effect increases the firm’s ex ante investment (so \( \theta \) is higher). If the potential damage \( D \) is low enough so that \( \hat{L} = C \), then the substitution effect always dominates, resulting in a decreasing \( \theta(L) \) curve. But if the potential damage is higher so that \( \tilde{L} > C \), then the output effect may dominate, resulting in either an increasing or an inverted U-shaped \( \theta(L) \) curve.

The above result reveals how ex ante investment varies with product liability. This relationship depends on environmental factors such as potential damage \( D \) (holding all other parameters of the model constant), as the nature of \( \theta(L) \) depends on how large \( D \) is.
We next illustrate Proposition 1 with two examples. For both examples, suppose that $\gamma$ is distributed uniformly on $[0,1]$.

**Example 1** Suppose that consumers’ values follow the uniform distribution on $[0,1]$. In addition, $k(\theta) = \theta^2/2$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where $M$ is sufficiently large (for example, $M > D$) so that $\bar{\theta} = 0.9$. Let $C = 0.5$ and $\beta = 1$.

1. If $D \leq 1.172$, then $\theta(L)$ decreases in $L$ for $L \in [C,D]$.
2. If $D > 1.172$: then there exists $\hat{L} < D$ such that $\theta(L)$ strictly increases in $L$ for $L \in [C,\hat{L})$ and strictly decreases in $L$ for $L \in (\hat{L},D]$. For instance, when $D = 2$, $\hat{L} = 0.787$.

In Example 1, the equilibrium $\theta$ is a decreasing function of $L$ when $D$ is relatively small ($D \leq \tilde{D} = 1.172$) but an inverted U-shaped function of $L$ when $D$ is above $\tilde{D}$. Example 2 below illustrates that the equilibrium $\theta$ may also monotonically increase in $L$.

**Example 2** Suppose that consumers’ value follows the uniform distribution on $[0,1]$. In addition, $k(\theta) = \theta^2/8$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where $M$ is sufficiently large (for example, $M > D$) so that $\bar{\theta} = 0.9$. Let $C = 0.5$ and $\beta = 1$.

1. If $D \leq 0.944$, then $\theta$ decreases in $L$ for $L \in [C,D]$ (i.e. the substitution effect dominates).
2. If $0.944 < D \leq 4.464$: then there exists $\hat{L} < D$ such that $\theta$ increases in $L$ for $L \in [C,\hat{L})$ and decreases in $L$ for $L \in (\hat{L},D]$.
3. If $D > 4.464$: then $\theta$ increases in $L$ for $L \in [C,D]$ (i.e., the output effect dominates).

In sum, this section has shown why and when there can be an increasing, decreasing, or non-monotone relationship between the firm’s ex ante quality investment and product liability. Increasing the firm’s liability may not always increase the ex ante quality investment or reduce the expected ex post social cost. This observation will have important implications for determining the socially optimal liability policy, which we next turn to.

4. PRIVATE V.S. SOCIAL INCENTIVES FOR PRODUCT LIABILITY

In this section, we examine how product liability $L$ affects firm profit, consumer surplus, and social welfare. These discussions will shed light on whether it is socially optimal to
impose full liability or partial liability for the firm. As in Section 3, for ease of exposition we assume $L \leq D$.

Define social welfare as $W = \Pi + U$, where $\Pi$ is the firm’s expected profit and $U$ is the aggregate consumer surplus. From the analysis in Section 3, we have

$$\Pi = \max_{\theta} Q(\theta, L)[F^{-1}(1 - Q(\theta, L)) - (1 - \theta)\Delta(L)] - k(\theta),$$

$$U = \int_{t}^{t} (v - t)dF(v) = \int_{t}^{t} [v - F^{-1}(1 - Q(\theta, L))] dF(v).$$

Note that consumer surplus only depends on total output $Q(\theta, L)$. Intuitively, the marginal consumer with $v = t$ is indifferent between purchasing and not purchasing the product. From the ex ante point of view, all consumers face the same expected harm if the product quality is low. When there are more sales, there is larger consumer surplus.

**Proposition 2** Firm profit is maximized under full liability $L = D$.

Thus, ex ante, the firm always prefers full liability, although ex post the firm would prefer no liability. Intuitively, if the firm bears only partial liability, its ex post actions may not be socially efficient. The unit ex post social cost from low quality would be larger. However, as discussed in Section 3, given $\theta$, the firm fully internalizes the social cost from low quality. Full liability allows the firm to create the intertemporal commitment to take efficient ex post actions that minimize expected unit social cost from low quality.

Given that the firm has market power, to minimize ex post social cost of low quality, the firm may reduce its output and therefore consumer surplus would be decreased. Note that the firm’s output depends on its ex ante quality investment. Our next Proposition establishes when consumer welfare is maximized with full or partial liability. The result depends importantly on condition $T1$ below, where $\tilde{\theta}$ is defined in Section 3.

$T1$: There is some $\theta \in (0, \tilde{\theta})$ such that $\frac{k'(\theta)}{k(\theta)} > 1 - \theta$.

**Proposition 3** (i) Suppose that condition $T1$ holds. Then there exist values $D_L < D_H$ such that consumer surplus is maximized under full liability if $D \geq D_H$, and consumer
surplus is maximized under partial liability if $D_L \leq D < D_H$. (ii) Suppose that $T1$ does not hold (i.e., for all $\theta \in (0, \tilde{\theta}], k'(\theta)/k''(\theta) \leq 1 - \theta$). Then consumer surplus is maximized under full liability.

To see the intuition for this consumer welfare result, notice first that consumer surplus is higher if there is more output. When the damage level $D$ is large enough, for any liability $L < D$, the output effect dominates the substitution effect: when $L$ increases, the firm takes more ex ante quality investment. Correspondingly, the expected unit social cost $(1-\theta)\Delta(L)$ becomes smaller and therefore the firm increases output even further. Thus, full liability maximizes output and consumer surplus. On the other hand, when $D$ is in some intermediate range, the substitution effect dominates the output effect: as $L$ increases, the firm takes less ex ante quality investment. Then the expected unit social cost $(1-\theta)\Delta(L)$ may become larger and thus output may decrease.

Condition $T1$, which holds if the cost of ex ante investment is "not too convex", can be understood intuitively as follows. Define a new function $u(\cdot)$ such that $u(1-\theta) = -k(\theta)$. It can be verified that $u(\cdot)$ is increasing and concave. Then condition $T1$ becomes: There exists $\theta \in (0, \tilde{\theta}]$ such that

\begin{equation}
    -(1-\theta)u''(1-\theta) < 1.
\end{equation}

The left-hand side of (11) can be interpreted as the Arrow-Pratt measure of "relative risk aversion" of the firm towards the probability of low quality. Therefore, Condition $T1$ says that the firm does not always have a "high" degree of "risk aversion" towards the probability of low quality. Consider the range $D_L \leq D < D_H$ where the substitution effect dominates the output effect. If Condition $T1$ holds, when $L$ increases, the marginal increase in $(1-\theta)$ is relatively large; but the marginal decrease in the ex post per unit cost $\Delta(L)$, which is independent of the risk aversion measure, can be small. Consequently, increasing liability $L$ might cause $(1-\theta)\Delta(L)$ to increase, which leads to smaller output and lower consumer surplus. On the other hand, if $T1$ does not hold, or the firm always has a relatively high

\footnote{We note that $D_L$ and $D_H$ are independent of $L$, and $Q > 0$ when $D = D_H$ so that the firm will indeed produce positive amount of output when $D$ is greater than $D_H$ but not too high.}
degree of risk aversion towards the probability of low quality, then when $L$ increases, the firm reduces ex ante investment but not by too much, and thus the marginal increase in $(1 - \theta)$ is small; but the marginal decrease in $\Delta(L)$ can be relatively large. In this latter case, it turns out that the overall effect from increasing liability $L$ would cause $(1 - \theta)\Delta(L)$ to decrease, which leads to larger output and higher consumer surplus. Condition $T_1$ is satisfied, for example, if consumers’ values follow the uniform distribution and $k(\theta) = a\theta^2$ for any $a \leq \frac{1}{8}$.

Proposition 3 suggests the following policy implications. If the firm is always highly "risk averse" towards the probability of low quality, in the sense that $T_1$ cannot hold, full liability is optimal for consumer welfare—increasing $L$ is effective in improving ex post efficiency but does not reduce ex ante investment much. Otherwise, either full or partial liability can be best for consumers, depending on how large the potential damage from low quality is: full liability is optimal if $D$ is high enough, but partial liability is optimal if $D$ is in some intermediate range.\(^{22}\)

Since the firm and the consumers sometimes have conflicting interests in their preferred liability rule, we next consider when full or partial liability maximizes social welfare. By the log concavity of $f(v)$, $H'(v) = d\{[1 - F(v)] / f(v)\} / dv \leq 0$ and $H''(v) \geq 0$. It follows that $h = \max_v[-H'(v)] = -H'(0)$. Our welfare result below refers to condition $T_2$, which like $T_1$ holds if the cost of ex ante investment is not too convex.

$T_2$: There exists $\theta \in (0, \hat{\theta})$ such that $\frac{k'(\theta)}{k''(\theta)} > (1 - \theta)(2 + h)$.

**Proposition 4** Suppose that condition $T_2$ holds. Then there exist values $\bar{D}_L < \bar{D}_H$ such that $[\bar{D}_L, \bar{D}_H] \subset [D_L, D_H]$: If $D \geq \bar{D}_H$, social welfare is maximized under full liability; if $\bar{D}_L \leq D < \bar{D}_H$, social welfare is maximized under partial liability.

Condition $T_2$, like $T_1$, can be viewed as saying that the firm is not always too "risk averse" towards the probability of low quality. The intuition for the role of $T_2$ in Proposition

\(^{22}\)Again, as discussed in Section 3, if $D$ is small enough ($D < C$), then the firm’s actions will be the same for all $L \leq D$. 

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4 is similar to that of T1 in Proposition 3. Consider the range $D_L \leq D < D_H$ where the substitution effect dominates the output effect. If condition T2 holds, an increase in $L$ causes a relatively large marginal reduction in $\theta$, or a relatively large marginal increase in $(1 - \theta)$; but the marginal reduction in $\Delta(L)$, which does not depend on the risk aversion measure, is small. In this case, increasing liability $L$ might cause $(1 - \theta)\Delta(L)$ to increase, which leads to smaller output and lower consumer surplus, and the decrease in consumer surplus may further dominate potential increases in firm profit. Although more restrictive than condition T1, T2 is also satisfied, for example, if consumers’ values follow the uniform distribution and $k(\theta) = a\theta^2$ for any $a \leq \frac{1}{5}$.

We illustrate Proposition 4 with an example that continues Example 2 in Section 3:

**Example 3** Suppose that consumers’ values follow the uniform distribution on $[0, 1]$. In addition, $k(\theta) = \theta^2/8$ if $\theta \leq 0.9$ and $k(\theta) = M$ if $\theta > 0.9$, where $M$ is sufficiently large (for example, $M > D$). That is, $\overline{b} = 0.9$. Let $\beta = 1$. Assume that $\gamma$ also follows the uniform distribution on $[0, 1]$.

Figure 1 illustrates how the socially optimal liability depends on the damage level $D$, where the red curve is defined by $C = D - D\sqrt{1 - \frac{0.944}{D}}$, the green curve is defined by $C = D - D\sqrt{1 - \frac{0.614}{D}}$, and the purple curve is from simulation such that no liability and full liability lead to the same social welfare for $D < 0.944$. Notice that full liability $L = D$ is more efficient in Range F where $D$ is sufficiently high; partial liability $L \in (C, D)$ is more efficient in Range P where $D$ is intermediate; and it is more efficient to impose no liability (or, equivalently, any $L < C$) for the firm in Range N. These results are consistent with the general predictions in Proposition 4.

In particular, let $C = 0.5$. Then full liability is more efficient when $D < 0.870$ or $D > 4.464$; partial liability $L \in (C, D)$ is more efficient when the damage level is intermediate ($0.944 < D < 4.464$); zero liability (or, equivalently, any $L < C$) is more efficient when $0.870 < D < 0.944$. 

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The above example also shows that social welfare may not always increase in liability. In particular, let $D = 1$ and $C = 0.5$. Then we have the following relationship between social welfare and liability: Social welfare is maximized under partial liability $L = 0.532$; and for $L \geq C = 0.5$,

$$W \begin{cases} 
\text{increases in } L & \text{if } L \in [0.5, 0.532] \\
\text{decreases in } L & \text{if } L \in (0.532, 0.695) \\
\text{increases in } L & \text{if } L \in (0.695, 1]
\end{cases}$$

Proposition 4 shows that the socially optimal liability depends on the potential damage level. When the potential damage is large enough, the output effect dominates the substitution effect of increasing liability. Therefore, when the firm bears larger liability, both output $Q$ and the ex ante investment $\theta$ would increase, which increases overall social welfare. Thus full liability is socially optimal. In contrast, when the potential damage is at intermediate levels, the substitution effect dominates the output effect. Therefore, the ex ante investment is lower under larger liability. Moreover, as long as the firm is not too "risk averse" towards the probability of low quality, the output can also decrease, which may reduce social welfare. Partial liability can then be more efficient than full liability.

To illustrate the relevance of the parameter values identified in Proposition 4, we consider the finding in Example 3 that, with $C = 0.5$, full liability is more efficient when $D/C >$
4.464/0.5 ≈ 8.93 but partial liability $L \in (C, D)$ is more efficient when the damage level is intermediate ($1.89 < D/C < 8.93$). For instance, in 2001, Ford and Firestone recalled about 18.5 million tires fitted on the Ford Explorer. The expected per unit cost was about $500. Similarly, in 2004, General Motors recalled more than 10.5 million vehicles in North America and the per unit recall cost was about $300. Potential damage from car accidents or malfunction ($D$) is likely to be much higher than the cost of remedial actions ($C$), or $D/C$ is likely to be higher than $4.464/0.5 ≈ 8.93$. Our result then suggests that full liability would be more efficient (than partial liability) in these cases. On the other hand, in 2007, Sanyo Electric in Japan planned to recall 1.3 million battery packs used on certain non-high-end mobile phones, due to the risk of excessive heat. The estimated per unit cost, $C$, was about $30$. If we estimate $D$ based on the market price of similar phones, it seems reasonable to assume that $D/C$ would be in the range of [4, 6], which is within $(1.89, 8.93)$, the range of $D/C$ for which our result suggests that partial liability would be more efficient than full liability.

There have been substantial debates on how product liability affects firms’ ex post remedial actions such as product recalls. Our findings suggest that a policy or liability rule should also consider its effects on output and the incentive for ex ante investment. The interactions between ex ante investment and ex post remedial actions are subtle and important. Although a larger product liability can enhance the ex post efficiency of remedial actions, it may reduce ex ante investment and output, which would lead to more deadweight loss. As we have shown, for a substantial range of damage levels, partial liability is more efficient than full liability. Thus the socially efficient liability rule depends on the potential damage levels. For products with large potential consumer damages relative to ex post remedial costs, full liability is more efficient than partial liability; for products with intermediate potential damages relative to ex post remedial costs, partial liability could be more efficient.

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23The total expected cost was approximately $3 billion (Legal Week, January 29, 2004).
24The total expected cost was approximately $300 million (The Globe and Mail, November 18, 2004).
25The total expected cost was approximately 4 billion Japanese Yen (Jiji Press Ticket Service, April 11, 2007).
In practice, government agencies often rate potential damage to consumers for each product defect. For example, the National Highway and Traffic Safety Administration (NHTSA) gives a hazardous rating for each recalled product; the US Food and Drug Administration (FDA) specifies different classes of product harm from food and medical equipment. Such information, together with considerations of firm costs of ex ante investment and ex post remedial actions, potentially can be used to test policy implications from this paper.\footnote{There is limited empirical literature related to costs of product recalls and consumer damage. Rupp and Taylor (2002) contain an interesting study of whether NHTSA or manufacturers initiated automobile recalls depending on hazardous ratings and potential costs of recalls.}

5. DISCUSSIONS

5.1 Product Warranty

The previous analysis assumes that the firm cannot commit to taking ex post remedial actions if it bears no liability. One way for the firm to create such intertemporal commitment is to offer warranties. The existing literature mainly views warranty as a signaling device for a firm’s unobservable quality choice or quality investment (e.g. Grossman, 1981; Cooper and Ross, 1985; Lutz, 1989).\footnote{Similarly, during sales, firms may offer return policies as signaling devices (Moorthy and Srinivasan, 1995). Return policies may also enhance risk sharing between a firm and its consumers (Che, 1996).} It typically considers a firm’s expected compensation to consumers when a quality problem arises, but does not address the possibility for the firm to take ex post remedial actions such as product recalls before consumers are harmed.

Suppose that the firm could issue a warranty which specifies compensation $L$ to a consumer when she has utility loss due to low quality. The firm can take ex post remedial actions after sales. As shown in Section 4, the firm prefers full warranty $L = D$, which provides commitment that the firm would take efficient ex post actions. Such a full warranty, if perfectly enforced, maximizes the firm’s profits.

In practice, firms often offer limited warranty, because a full warranty may unduly reduce consumer care in using the product. Our findings in Section 4 suggest that a limited warranty can sometimes also be what consumers prefer, because it can lead to higher ex...
ante quality investment and output, which benefit consumers.

Another potential welfare loss from full warranty is that consumers may not comply with the firm’s ex post remedial activities. On one hand, with full warranty, consumers’ negligence would reduce the effectiveness of the firm’s ex post activities; on the other hand, it may induce the firm to make more ex ante investment. The next subsection provides more general discussions on whether consumers’ negligence should be considered in determining the firm’s liability.

5.2 Consumer Negligence

We now consider consumer negligence in two dimensions: how negligence rules affect consumers’ cooperation with the firm’s ex post remedial actions, and what if consumers can also take precaution effort.

Our main model assumes that \( \beta \) is exogenously given. In practice, the effects of ex post remedial actions may hinge on consumers’ awareness or incentives to comply with the firm’s actions. Consumers’ incentives can be affected by liability rules. For example, courts may use either strict liability or negligence rules.\(^{28}\) Under strict liability, the firm bears the same liability \( L \) no matter whether it has taken ex post remedial actions or not. Under negligence rules, the firm’s liability may be reduced if the firm has taken ex post remedial actions and consumers get informed but do not comply.\(^{29}\) Under negligence rules, consumers have more incentives to comply with the firm’s ex post activities. We now illustrate how the form of liability rule, strict liability vs. negligence rule, affects the firm’s ex post incentive for remedial actions, particularly product recalls, as well as overall efficiency.

Assume that, under strict liability as analyzed in Section 3, the ex post remedial action can fix a proportion \( \beta \) of the sold product; under the negligence rule, the ex post action can

\(^{28}\)There is a large literature comparing strict liability and negligence rules. For examples, see Brown (1973), Green (1976), Shavell (1980), Rubinfeld (1987), Emons (1990), Emons and Sobel (1991), Bar-Gill and Ben-Shahar (2003).

\(^{29}\)The choice between strict liability and negligence rules may affect the firm’s incentives to take ex-post remedies, for example, as discussed in Hua (2011).
fix a proportion $\beta_N > \beta$ of the sold product.

Under the negligence rule, the firm no longer bears liability if consumers do not comply with ex post remedial activities. Hence, the firm will take ex post remedial actions if and only if $C\beta_N Q < \gamma LQ$. Given low product quality, the expected ex post social costs per unit of the product are defined as

$$\Delta_N = \int_0^{C\beta_N L} \gamma DdG(\gamma) + \int_{C\beta_N L}^{1} [\beta_N C + (1 - \beta_N)\gamma D]dG(\gamma).$$

In contrast, as shown in Section 3, under strict liability, the firm will take ex post actions if and only if $C\beta Q < \gamma \beta LQ$. Given low product quality, the expected ex post social costs per unit of the product are defined as

$$\Delta_S = \int_0^{C\beta} \gamma DdG(\gamma) + \int_{C\beta}^{1} [\beta C + (1 - \beta)\gamma D]dG(\gamma).$$

When the legal system is changed from strict liability to the negligence rule, the firm is more likely to take ex post remedial actions, since $\frac{C\beta_N L}{L} < \frac{C}{L}$. However, this change may not necessarily increase ex post efficiency. To see this, suppose that $\beta_N D < L$. Then $\frac{C\beta_N}{L} < \frac{C}{L}$, that is, the negligence rule both motivates the firm to take too many ex post remedial activities relative to socially desired, and increases the effectiveness of such ex post activities. In this case, negligence rules may lead to either higher or lower ex post social costs than strict liability.

If $\beta_N D \geq L$, then $\frac{C}{D} \leq \frac{C\beta_N}{L} < \frac{C}{L}$. In this case, negligence rules lead to lower ex post social costs than strict liability, i.e., $\Delta_N < \Delta_S$. However, according to Proposition 4, for intermediate damage levels, reducing ex post social costs may decrease social welfare. Therefore, strict liability can be socially more efficient. The full-fledged comparison between strict liability and negligence rules is ambiguous in this general framework. However, this discussion illustrates that the choice between strict liability and negligence rules should also take firms’ ex ante quality investment and output into consideration. If negligence rules increase ex post efficiency but reduce firms’ ex ante quality investment and output significantly, then it may be more efficient to impose strict liability or set a higher standard.
for evidence in determining consumers’ negligence.

Next consider the possibility that consumers can take precaution after the firm’s ex ante investment and after sales are made. For certain products such as food and toys, negligence rules would induce firms to make more announcements and motivate consumers to take precaution (such as stop using products).\(^{30}\) For other products with larger consumer values, even though consumer precaution is still possible, firms often have more skills or information in fixing the products.

Compared to strict liability, negligence rules motivate consumers to take more precaution if the product turns out to have low quality. Then there also exist both the substitution effect and the output effect: (1) given the output, more consumer precaution reduces the firm’s ex ante investment incentive; (2) however, more consumer precaution leads to larger output, which in turn increases ex ante investment. In light of these two conflicting effects, even if consumers can take precaution, strict liability can sometimes be more efficient than negligence rules.

5.3 Punitive Damage Compensation

Our main analysis in Sections 3 and 4 focuses on the scenario with \( L \leq D \). In practice, sometimes courts may impose punitive damage compensation so that \( L > D \). If there is punitive compensation, as shown in Lemma 1, the firm would take ex post actions more frequently than socially desired. In addition, the following proposition shows that, as long as punitive damage compensation cannot be too large (perhaps because firms can resort to bankruptcy protection), partial liability \( L < D \) can lead to the same social welfare as punitive damage compensation.

**Proposition 5**  (1) If \( \int_0^1 (C - \gamma D)dG(\gamma) \leq 0 \), then for any punitive damage compensation \( L > D \), there exists \( L' < D \) which leads to the same social efficiency.  (2) If \( \int_0^1 (C - \gamma D)dG(\gamma) > 0 \), then there exists \( \bar{L} > D \) such that for any punitive damage compensation \( L \in (D, \bar{L}] \), there exists \( L' < D \) which leads to the same social efficiency.

\(^{30}\)See Spier (2011) for more discussions.
Intuitively, the firm’s liability influences social welfare only through the change of ex post unit social cost, $\Delta$. For any punitive damage compensation $L \in (D, \overline{L}]$, we can always find a liability level $L \leq D$ which leads to the same ex post unit social cost and correspondingly, the same social welfare.

On the other hand, if punitive damage compensation is sufficiently large ($L > \overline{L}$), it causes the firm to take much more ex post remedial actions than socially desired. The ex post social cost would become large, which may motivate the firm to take more ex ante quality investment. Thus, it is possible that very large punitive damage compensation $L > \overline{L}$ may be more efficient than $L \leq D$. However, in practice, it is often difficult to impose very large punitive damage compensation, especially when firms can seek bankruptcy protection or the legal enforcement is not perfect.

5.4 Alternative Quality Investment or Ex Post Remedial Technology

We now discuss the robustness of our results with respect to alternative assumptions on ex ante investment and ex post remedial actions.

First, our analysis has assumed that the ex ante investment cost is convex while the ex post remedial cost is linear. In essence, our analysis is about how the firm optimally balances between the two forms of investment. Thus, what seems crucial is the relative convexity of the two cost functions, and the linearization of the ex post remedial cost is primarily to simplify analysis. Our main insights, particularly the trade-off between the substitution and output effects, can still hold if the ex post remedial cost is also convex in $Q$. To illustrate this, suppose that $\beta = 1$ and the ex post total remedial cost is $C(Q) = \frac{1}{2} \phi Q^2$ for $\phi > 0$. In addition, suppose that $\gamma$ follows the uniform distribution on $[0, 1]$.

In the ex post period, if product quality is low, the firm will take remedial actions if and only if $C(Q) < \gamma LQ$, or, equivalently, $\gamma > C(Q)/LQ = \frac{\phi Q}{2L}$.

Given low product quality, the expected ex post social cost per unit of output becomes

$$\Delta = \Delta(L, Q) = \int_0^{\frac{\phi Q}{2L}} \gamma D d\gamma + \int_{\frac{\phi Q}{2L}}^1 \frac{C(Q)}{Q} d\gamma = \int_0^{\frac{\phi Q}{2L}} \gamma D d\gamma + \int_{\frac{\phi Q}{2L}}^1 \frac{\phi Q}{2} d\gamma.$$
It can be verified that \( \Delta(L, Q) \) decreases in \( L \) and increases in \( Q \). Similar to the analysis in Section 3, given the ex ante quality investment \( \theta \), the firm chooses \( Q \) to maximize its profit

\[
\pi(\theta) = \max_{Q \leq 1} Q [p - (1 - \theta)x]
\]

(13)

\[
= \max_{Q \leq 1} Q[F^{-1}(1 - Q) - (1 - \theta)\Delta(L, Q)].
\]

The first order condition is

\[
F^{-1}(1 - Q) - \frac{Q}{f[F^{-1}(1 - Q)]} = (1 - \theta)\Delta(L, Q) + Q(1 - \theta)\frac{\partial \Delta(L, Q)}{\partial Q}.
\]

In particular,

\[
\frac{\partial \Delta(L, Q)}{\partial Q} = \frac{\phi}{2} + \frac{\phi^2 Q}{2L} \frac{D}{2L - 1}.
\]

For \( L \leq D \), \( \frac{\partial \Delta(L, Q)}{\partial Q} = \frac{\phi^2 Q}{2L} (1 - \frac{D}{L}) \leq 0 \). Then, as in Section 3, it can be verified that given \( \theta \), \( Q \) also increases in \( L \) here. The firm’s optimal ex ante investment satisfies the following condition:

\[
Q\Delta(Q, L) - k'(\theta) = 0,
\]

from which we can identify the substitution effect and the output effect. Given \( Q \), increasing liability \( L \) reduces \( \Delta \), which reduces the firm’s incentives for ex ante investment. On the other hand, a marginal increase in \( L \) leads to larger output \( Q \), which in turn may increase \( Q\Delta \) (given \( L \)). That is, the output effect motivates the firm to take more ex ante investment. In sum, the trade-off between these two effects can still be present when the ex post remedial cost is convex.\(^{31}\)

Second, in the main model, the firm’s ex ante quality investment \( \theta \) affects the probability for the product to have low quality. Once the product is of low quality, there is uncertainty concerning whether a consumer will be harmed, and the (conditional) probability of harm,

\(^{31}\)A sufficient condition for the existence of these effects is that, given \( \theta \), the firm’s optimal output \( Q \) increases in \( L \). For the general convex cost function \( C(Q) \), this condition holds as long as \( \frac{\partial \Delta(L, Q)}{\partial Q} \leq 0 \).
Our analysis on the relationship between product liability and ex ante investment can be extended to situations where the firm’s ex ante investment \( \theta \) affects the distribution of \( \gamma \).

In particular, suppose that the firm can make ex ante investment \( \theta \) and the likelihood for consumers to be harmed, \( \gamma \), follows a distribution \( G(\gamma \mid \theta) \). After sales, the firm learns the realization of \( \gamma \) and then decides whether to take ex post remedial actions. Assume \( G_\theta(\gamma \mid \theta) \geq 0 \). That is, \( G(\gamma \mid \theta) \) first-order stochastically dominates \( G(\gamma \mid \theta') \) for any \( \theta < \theta' \). With \( L \leq D \), it is easy to verify that the firm’s optimal ex ante quality investment satisfies

\[
Q(\theta, L)\Delta(\theta, L) - k'(\theta) = 0.
\]

Similar to the analysis in Section 3, when the firm’s liability \( L \) increases, there are again two conflicting effects on ex ante quality investment. On one hand, given the output \( Q \), there is a substitution effect between ex ante quality investment and ex post remedial actions. On the other hand, there is an output effect: the lower ex post unit social cost \( \Delta \) under a higher \( L \) may lead to larger output \( Q \), which in turn increases the firm’s investment \( \theta \). Therefore, as in our main model, the firm’s optimal ex ante investment may increase, decrease, or be non-monotone in its product liability.

### 5.5 Market Reputation and Product Liability

To identify the impact of product liability on a firm’s ex ante quality investment and ex post actions in a most transparent way, we have considered a highly stylized model that abstracts away from other potentially important factors, especially repeated interactions and reputation.

There is a limited literature on the interaction between market reputation and product liability. Polinsky and Shavell (2010) suggest that these two mechanisms are substitutes to

\[32\text{For example, before sales, the firm may make R&D investment to have safer product design. However, there is still non-trivial probability for the product to have design defect. Given the design defect, the likelihood for consumers to be harmed depends on the nature of the defect as well as consumers’ usage.}\]
each other. If reputation concern works well in motivating firms to take safety investment, then product liability may not be important. However, regarding ex post remedial actions, there are many situations where reputation effect may not work well. For example, consumers may not be informed about the harm suffered by previous consumers; or, even if they are informed, it may not be clear whether the harm was caused by the product or by other factors. Also, firms such as automobile manufacturers may no longer sell the old model of product which caused harm to previous consumers. In these and many other situations, reputation concerns would not fully solve the time inconsistency problem identified in this paper: the firm might not have sufficient incentives to take ex post remedial actions. Also, the firm might not have enough incentives to take ex ante investment. Product liability could then play an important role by providing more incentives. What our analysis has shown, however, is that this role is subtle: it works though the interactions between ex ante investment and ex post remedies and through the trade-off between the substitution and output effects.

Competition can also affect the roles played by product liability and reputation. With competition, a firm may have more concern that it will lose customers if it does not take proper ex post actions to fix product defects. The need for product liability may then be reduced. But competition could also motivate firms to market a product prematurely in order to preempt competition, which may increase the need for product liability. It would be interesting for future research to study how competition and product liability interact to affect incentives for product safety investment and for ex post remedial actions.

Furthermore, our analysis can have implications for the interactions between reputation concerns and other policies such as those that increase information disclosure. For example, when FDA imposes requirements to increase the effectiveness of information disclosure about product harm, firms would have greater reputation concern for future sales, motivating them to take more product recalls. More product recalls, however, can have both a substitution effect and an output effect, as identified in our analysis, which can have opposing effects on the firm’s incentive for ex ante quality investment. Hence, similar to changes in product liability, policy changes on information disclosure and the corresponding
changes in reputation concerns may not always increase a firm’s quality investment as well as consumer and social welfare.

6. CONCLUSION

This paper has studied how product liability affects product quality (safety) as well as consumer and social welfare. We find that the interactions between a firm’s ex post remedial actions for low product quality and its incentive for ex ante quality investment have important implications for the effects of product liability. Higher liability increases ex post remedial actions such as product recalls, and this can in turn reduce the incentive for ex ante investment. On the other hand, higher liability increases consumer demand for the product, which in turn increases the firm’s incentive for ex ante quality investment. The presence of these two opposing effects, the substitution effect and the output effect, implies that the ex ante quality investment may not monotonically increase in product liability—the relationship may be an inverted U-shaped curve. While full liability maximizes profit since it allows the firm to make the intertemporal commitment for ex post remedial actions, it may not be optimal for consumers when it reduces ex ante quality investment and output by the firm. Full product liability tends to be socially optimal when the potential consumer loss from low quality is sufficiently high; otherwise partial liability can be socially optimal.

There are several directions for further research. For example, given the trade-offs between the substitution and output effects, firms’ ex ante investment and ex post remedial actions could be either substitutes or complements. It would be useful to empirically verify which effect would dominate under different liability rules. Also, as mentioned earlier, it would be desirable to study the potential trade-offs involved and the implications for product liability rules under competition, and to explore how reputation concern interacts with liability rules and how they jointly affect firms’ ex ante investments and ex post actions. Furthermore, it is desirable in future research to relax our assumption that consumers can observe the firm’s ex ante investment. When such investment is the firm’s private information, the firm may
use pricing and other strategies to signal its quality investment, and product liability may then have additional implications for the interactions between the firm’s ex ante investment and ex post activities.

APPENDIX

**Proof of Lemma 3**: If the firm cannot make any ex post remedial action, then given low product quality, the firm’s expected ex post cost per unit of output is

\[ x = \int_0^1 \gamma L dG(\gamma) \]  

whereas the expected ex post loss for any consumer using a low-quality product is

\[ y = \int_0^1 \gamma (D - L) dG(\gamma) \]  

Thus, given low product quality, the expected ex post social cost per unit of output is given by

\[ \Delta \equiv x + y = \int_0^1 \gamma D dG(\gamma) \]  

Therefore, liability \( L \) does not affect \( \Delta \). Given the ex ante quality investment \( \theta \), the firm chooses \( Q \) to maximize its profit

\[ \pi(\theta) = \max_{Q \leq 1} Q [p - (1 - \theta)x] = \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)(x + y)] \]

\[ = \max_{Q \leq 1} Q [F^{-1}(1 - Q) - (1 - \theta)\Delta]. \]

Since \( L \) does not affect \( \Delta \), given the above expression, the firm’s optimal sales \( Q \) is independent of liability \( L \). Correspondingly, liability does not change the firm’s ex ante investment. Q.E.D.
Proof of Proposition 1: According to Lemma 1, given \( L \in [C, D] \), \( \Delta \) strictly decreases in \( L \). Also, note that liability \( L \) affects \( Q \) only through the change of ex post social cost \( \Delta \). Therefore, to simplify the notation, define \( Q(\theta, \Delta(L)) = Q(\theta, L) \). Thus, the firm’s optimal ex ante investment satisfies either \( \theta = \overline{\theta} \) or

\[
Q(\theta, \Delta) \Delta - k'(\theta) = 0.
\]

This allows us to first consider how the change of \( \Delta \) affects ex ante investment and prove the following claim.

Claim: There exists a unique cut-off \( \Delta_c \) such that \( \frac{d\theta}{d\Delta} > 0 \) when \( \Delta_c < \Delta \).

Given \( \theta \), the firm’s optimal output satisfies (9), from which we obtain

\[
Q_\theta'(\theta, \Delta) = \frac{\partial Q}{\partial t} \frac{\partial t}{\partial \Delta} = -f(t) \frac{1 - \theta}{1 - H'(t)}.
\]

The optimal \( \theta \) satisfies either \( \theta = \overline{\theta} \) or condition (19).

First, when \( \Delta = 0 \), condition (19) implies that \( \theta(\Delta = 0) = 0 \). (If there is no ex post social cost, the firm would not make any ex ante investment.) When \( \Delta \) goes to infinity, condition (9) and (8) imply \( Q = 0 \), which also implies \( \theta = 0 \). Therefore \( d\theta/d\Delta > 0 \) when \( \Delta \) is sufficiently small but \( \theta \) eventually decreases in \( \Delta \) when \( \Delta \) is sufficiently large. It suffices to consider the following two cases.

1. Consider the case with interior solution: There exists some \( \Delta_c \) at which \( d\theta/d\Delta = 0 \). From (19), we have

\[
\frac{d\theta}{d\Delta} = -\frac{Q(\theta, \Delta) + \Delta Q_\theta'(\theta, \Delta)}{\Delta Q_\theta'(\theta, \Delta) - k''(\theta)}.
\]

Note that the second order condition

\[
\Delta Q_\theta'(\theta, \Delta) - k''(\theta) \leq 0
\]

for interior solutions \( \theta \in (0, \overline{\theta}) \). Thus, the sign of \( \frac{d\theta}{d\Delta} \) is determined by that of

\[
Q(\theta, \Delta) + \Delta Q_\theta'(\theta, \Delta).
\]
In particular, \( Q(\theta, \Delta) = 1 - F(t) \) by definition. Thus,

\[
Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta) = 1 - F(t) - \Delta f(t) \frac{1 - \theta}{1 - H'(t)}.
\]

Equivalently,

\[
\frac{Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta)}{f(t)} = \frac{1 - F(t)}{f(t)} - \Delta \frac{1 - \theta}{1 - H'(t)} = H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)}.
\]

Therefore, the sign of \( \frac{\partial}{\partial \Delta} \) is the same as that of \( H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)} \). Since by assumption \( f(v) \) is log-concave, \( H'(t) \leq 0 \) and \( H''(t) \geq 0 \).

Suppose that, given a particular \( \Delta \), \( \frac{\partial}{\partial \Delta} \leq 0 \), or equivalently, \( H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)} \leq 0 \). When \( \Delta \) increases marginally, \( \theta(\Delta) \) cannot increase. This implies that \( (1 - \theta)\Delta \) would increase. According to condition (9), if \( (1 - \theta)\Delta \) increases, \( t \) must increase as well. Correspondingly, \( H(t) \) and \( 1 - H'(t) \) would decrease. Hence \( H(t) - \frac{(1 - \theta)\Delta}{1 - H'(t)} \) would decrease and become even more negative. Therefore, if we start at some \( \Delta \) such that \( \frac{\partial}{\partial \Delta} \leq 0 \), for any larger \( \Delta \), \( \frac{\partial}{\partial \Delta} < 0 \). This implies that there must be a unique cut-off \( \hat{\Delta} \) such that \( \frac{\partial}{\partial \Delta} \geq 0 \) when \( \Delta \leq \hat{\Delta} \).

(2) Now consider the case with corner solution or discontinuity in \( \theta \): that is, there exists some \( \hat{\Delta} \) at which \( \theta = \overline{\theta} \). Since \( \theta = \overline{\theta} \), increasing \( \Delta \) marginally from \( \hat{\Delta} \) cannot increase the firm’s ex ante investment \( \theta \) further. Therefore, according to condition (9), \( t \) must increase, or equivalently, \( Q(\overline{\theta}, \Delta) \) must decrease. When \( \Delta \) increases further, \( Q(\overline{\theta}, \Delta) \) decreases and gets closer to zero, so that \( Q(\overline{\theta}, \Delta) \Delta - k'(\overline{\theta}) \) eventually becomes negative, that is, \( \frac{\partial}{\partial \Delta} \leq 0 \). The rest of the proof is similar to that in part (1).

Given the above claim, we proceed to prove Proposition 1. According to Lemma 1, \( \Delta \) decreases in \( L \) given \( L \in [C, D] \). Let \( \hat{L} \) be such that

\[
\hat{L} = \begin{cases} 
    \hat{L} & \text{if there exists } \hat{L} \in (C, D) \text{ such that } \Delta(\hat{L}) = \hat{\Delta} \\
    D & \text{if } \Delta(D) \geq \hat{\Delta} \\
    C & \text{if } \Delta(C) \leq \hat{\Delta}
\end{cases}
\]

Then, when \( L < \hat{L} \), \( \Delta(L) > \hat{\Delta} \). Within this range, a higher liability \( L \), through decreasing \( \Delta \), increases \( \theta \). Therefore, \( (1 - \theta)\Delta \) decreases in \( L \). According to condition (9), \( t - H(t) = (1 - \theta)\Delta \), where \( t = F^{-1}(1 - Q) \), it then follows that \( Q(\theta, \Delta(L)) \) increases in \( L \). In contrast, if \( \hat{L} < D \), \( \Delta(L) < \hat{\Delta} \) when \( L > \hat{L} \), in which case \( \theta(L) \) decreases in \( L \).
Finally, Lemma 1 has shown that $\Delta$ increases in $D$ given $L$. Define $\tilde{D}$ such that $\Delta|_{L=C, D=\tilde{D}} = \tilde{\Delta}$. It is easy to verify that $\tilde{D}$ always exists and the corresponding output is positive. If $D < \tilde{D}$, then for any $L \leq D$, $\Delta(L) \leq \Delta(C) \leq \tilde{\Delta}$. Therefore, $\tilde{L} = C$. If $D > \tilde{D}$, then $\Delta(C) > \tilde{\Delta}$. Therefore, $\tilde{L} \in (C, D]$. Q.E.D.

**Proof of Proposition 2:** The firm’s optimal investment satisfies

$$\max_{\theta} Q(\theta, L)[F^{-1}(1 - Q(\theta, L)) - (1 - \theta)\Delta(L)] - k(\theta).$$

For any $\theta$ and $Q$, the objective function is higher with lower $\Delta(L)$. Therefore, the maximal profit also decreases in $\Delta$. When $L = D$, $\Delta$ is the lowest and $\Pi$ is the highest. Q.E.D.

**Proof of Proposition 3:** To simplify the use of notation, in this proof, define $Q(\theta, \Delta(L)) = Q(\theta, L)$. The firm’s optimal output is determined by

$$t - \frac{1 - F(t)}{f(t)} = t - H(t) = (1 - \theta)\Delta(L),$$

where $t = F^{-1}(1 - Q)$. The optimal quality investment $\theta$ satisfies either $\theta = \bar{\theta}$ or the following condition

$$Q(\theta, \Delta)\Delta(L) - k'(\theta) = 0$$

According to Proposition 1, when $\Delta \geq \tilde{\Delta}$, $\theta$ weakly decreases in $\Delta$. Therefore, when $\Delta$ increases, $(1 - \theta)\Delta$ would increase. Correspondingly, $Q(\theta, \Delta)$ would decrease. When $\Delta < \tilde{\Delta}$, $\theta$ increases in $\Delta$ and therefore $(1 - \theta)\Delta$ may increase or decrease in $\Delta$. Similar to the proof of Proposition 1, we have

$$Q'_\Delta(\theta, \Delta) = -f(t)\frac{1 - \theta}{1 - H'(t)}$$

and

$$Q'_{\theta}(\theta, \Delta) = \frac{\Delta f(t)}{1 - H'(t)}.$$ 

Therefore,

$$\frac{d[(1 - \theta)\Delta]}{d\Delta} = (1 - \theta) - \Delta \frac{d\theta}{d\Delta}$$

$$= (1 - \theta) + \Delta \frac{Q(\theta, \Delta) + \Delta Q'_\Delta(\theta, \Delta)}{\Delta Q'_{\theta}(\theta, \Delta) - k''(\theta)} = (1 - \theta) + \Delta \frac{(1 - F(t)) - \Delta f(t)\frac{1 - \theta}{1 - H'(t)}}{\Delta^2 f(t) - k''(\theta)}.$$
Note that the second order condition implies $\Delta Q'_\theta(\theta, \Delta) - k''(\theta) \leq 0$ for interior solutions. Thus, $(1 - \theta) + \Delta \frac{(1 - F(t)) - \Delta f(t) \frac{1 - \theta}{1 - H'(t)}}{\Delta f(t) \frac{1 - \theta}{1 - H'(t)} - k''(\theta)} < 0$ is equivalent to
\[
\Delta(1 - F(t)) - \Delta^2 f(t) \frac{1 - \theta}{1 - H'(t)} > (1 - \theta)(k''(\theta) - \Delta^2 f(t) \frac{1}{1 - H'(t)}), \text{ or}
\]
\[
\Delta(1 - F(t)) > (1 - \theta)k''(\theta).
\]
Given
\[
Q(\theta, \Delta) \Delta - k'(\theta) = 0,
\]
the above condition becomes $k'(\theta) > (1 - \theta)k''(\theta)$. Therefore, as long as Condition T1 holds, there exists a non-empty set $[\Delta_L, \Delta_H]$ (where $\Delta_H \leq \hat{\Delta}$) such that, for any $\Delta \in [\Delta_L, \Delta_H]$, $\frac{d[(1 - \theta)\Delta]}{d\Delta} < 0$ and therefore $Q(\theta, \Delta)$ increases in $\Delta$. According to Lemma 1, $\Delta$ increases in $D$. Define $D_L$ such that $\Delta_L = \Delta(L = D_L)$ and $D_H$ such that $\Delta_H = \Delta(L = D_H)$.

Since $\Delta_H \leq \hat{\Delta}$, the equilibrium output is positive for any $D < D_H$. Correspondingly, when $D \geq D_H$, $Q$ and $U$ decrease in $\Delta$ (or equivalently increase in $L$), and therefore consumer surplus is maximized under $L = D$. In contrast, if $D_L \leq D < D_H$, then at $L = D$, $Q$ increases in $\Delta$ (or equivalently decrease in $L$). Therefore, consumer surplus is maximized under partial liability $L \in [C, D)$. Q.E.D.

**Proof of Proposition 4:** To simplify the use of notation, in this proof, define $Q(\theta, \Delta(L)) = Q(\theta, L)$. First, when $\Delta \geq \hat{\Delta}$, The proofs of Propositions 2 and 3 imply that both the firm and consumers prefer $L = D$. Therefore, full liability is socially optimal. In the following analysis, suppose $\Delta < \hat{\Delta}$.

Note that
\[
\frac{d\Pi}{d\Delta} = -(1 - \theta)Q(\theta, \Delta)
\]
and
\[
\frac{dU}{d\Delta} = \frac{dU}{dt} \frac{dt}{d\Delta} = -(1 - F(t)) \frac{dt}{d\Delta} = -Q(\theta, \Delta) \frac{dt}{d\Delta}.
\]

Given $t - \frac{1 - F(t)}{f(t)} = t - H(t) = (1 - \theta)\Delta$,
\[
\frac{dt}{d\Delta} = \frac{1}{1 - H'(t)} \frac{d[(1 - \theta)\Delta]}{d\Delta} = \frac{1}{1 - H'(t)} \left\{(1 - \theta) - \Delta \frac{(1 - F(t)) - \Delta f(t) \frac{1 - \theta}{1 - H'(t)}}{k''(\theta) - \Delta^2 f(t) \frac{1 - \theta}{1 - H'(t)}} \right\}.
\]
Note that

$$\Delta(1 - F(t)) = \Delta Q(\theta, \Delta) = k'(\theta).$$

Then we have

$$\frac{dU}{d\Delta} = Q(\theta, \Delta) \left\{ \frac{k'(\theta) - \Delta^2 f(t) \frac{1-\theta}{1-H'(t)}}{k''(\theta)(1-H'(t)) - \Delta^2 f(t)} - \frac{1 - \theta}{1-H'(t)} \right\}.$$

If \(\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0\), then increasing \(\Delta\) would increase social welfare.

$$\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0$$

is equivalent to

$$\frac{k'(\theta) - \Delta^2 f(t) \frac{1-\theta}{1-H'(t)}}{k''(\theta)(1-H'(t)) - \Delta^2 f(t)} - \frac{1 - \theta}{1-H'(t)} > 1 - \theta.$$ 

The second order condition of (19) implies

$$k''(\theta)(1-H'(t)) - \Delta^2 f(t) > 0.$$ 

Therefore, the above inequality is equivalent to

$$k'(\theta) > (1 - \theta)k''(\theta)[2 - H'(t)] - (1 - \theta)\Delta^2 f(t).$$ 

Therefore, given condition T2, there exists a non-empty set \([\Delta_L, \Delta_H]\) (where \(\Delta_H \leq \Delta\)) such that the above inequality holds. In this case, for any \(\Delta \in [\Delta_L, \Delta_H]\), \(\frac{dU}{d\Delta} + \frac{d\Pi}{d\Delta} > 0\).

According to Lemma 1, \(\Delta\) increases in \(D\). Define \(\bar{D}_L\) such that \(\bar{\Delta}_L = \Delta(L = \bar{D}_L)\) and \(\bar{D}_H\) such that \(\bar{\Delta}_H = \Delta(L = \bar{D}_H)\). When \(D \geq \bar{D}_H\), \(W = U + \Pi\) decrease in \(\Delta\) (or equivalently increase in \(L\)), and therefore social welfare is maximized under \(L = D\). If \(\bar{D}_L \leq D < \bar{D}_H\), then at \(L = D\), \(W\) increases in \(\Delta\) (or equivalently decrease in \(L\)). Thus social welfare is maximized under \(L < D\). Note that condition T2 implies condition T1 but the reverse might not be true. Therefore, it can be verified that \([\bar{D}_L, \bar{D}_H] \subset [D_L, D_H]\). Q.E.D.

**Proof of Proposition 5:** According to Lemma 1, \(\Delta\) decreases in \(L\) when \(C \leq L \leq D\) and increases in \(L\) when \(L > D\). Note that \(\Delta(L = C) = \int_0^L \gamma D dG(\gamma) > \Delta(L = D)\). When \(L\) goes to infinity, \(\Delta(L \to \infty) = \int_0^L [\beta C + (1 - \beta) \gamma D] dG(\gamma)\). (1) If \(\int_0^L (C - \gamma D) dG(\gamma) \leq 0\), then \(\Delta(L \to \infty) \leq \Delta(L = C)\). In this case, for any punitive damage compensation \(L >
there exists $L' < D$ such that $\Delta(L) = \Delta(L')$. (2) If $\int_0^1 (C - \gamma D) dG(\gamma) > 0$, then $\Delta(L \to \infty) > \Delta(L = C)$. Given continuity, there exists $\overline{L} > D$ such that $\Delta(L = \overline{L}) = \Delta(L = C)$. Therefore, for any $L \in (D, \overline{L}]$, there exists $L' < D$ such that $\Delta(L) = \Delta(L')$. Correspondingly, $\theta(L) = \theta(L')$ and $W(L) = W(L')$. Q.E.D.
REFERENCE


